CHAPTER 7

Coordinates Geometry

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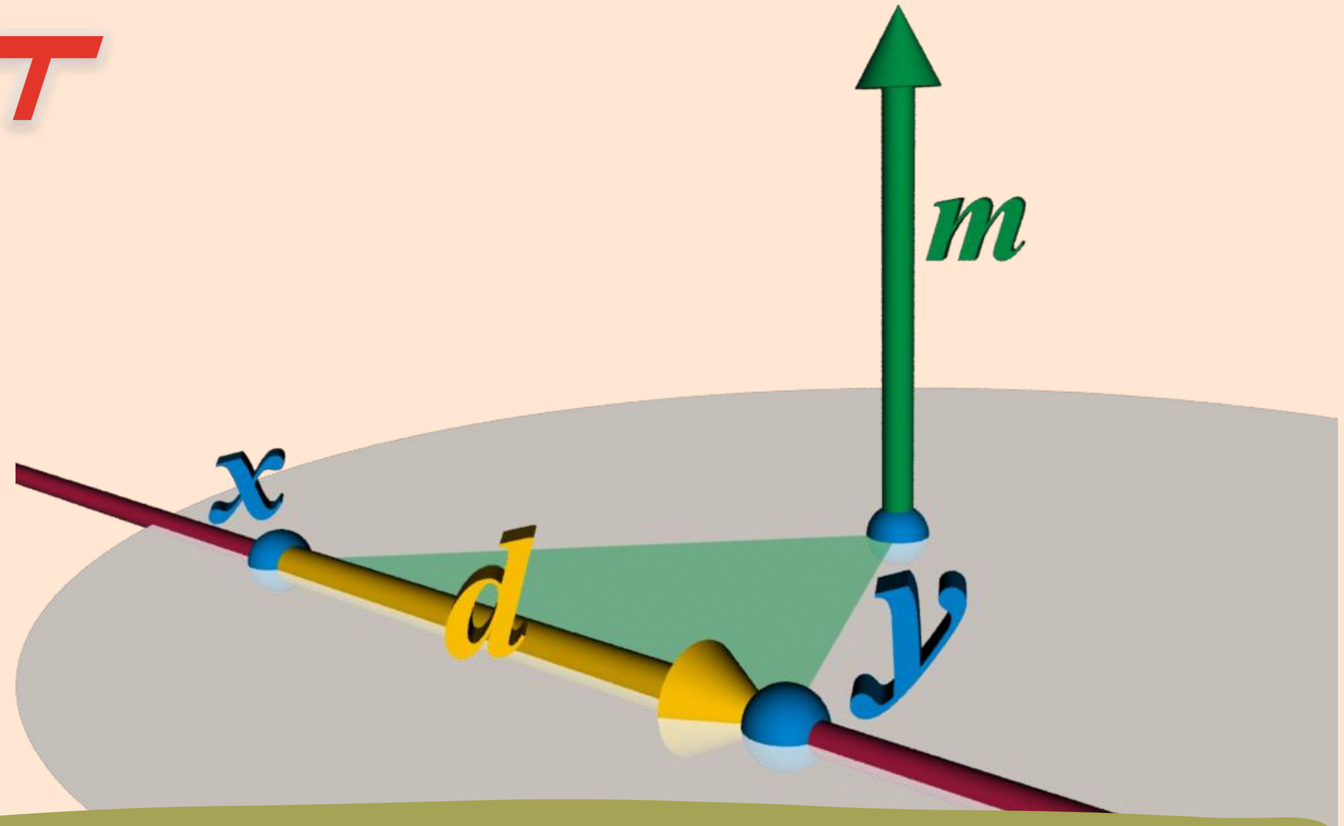
MRSM TUMPAT

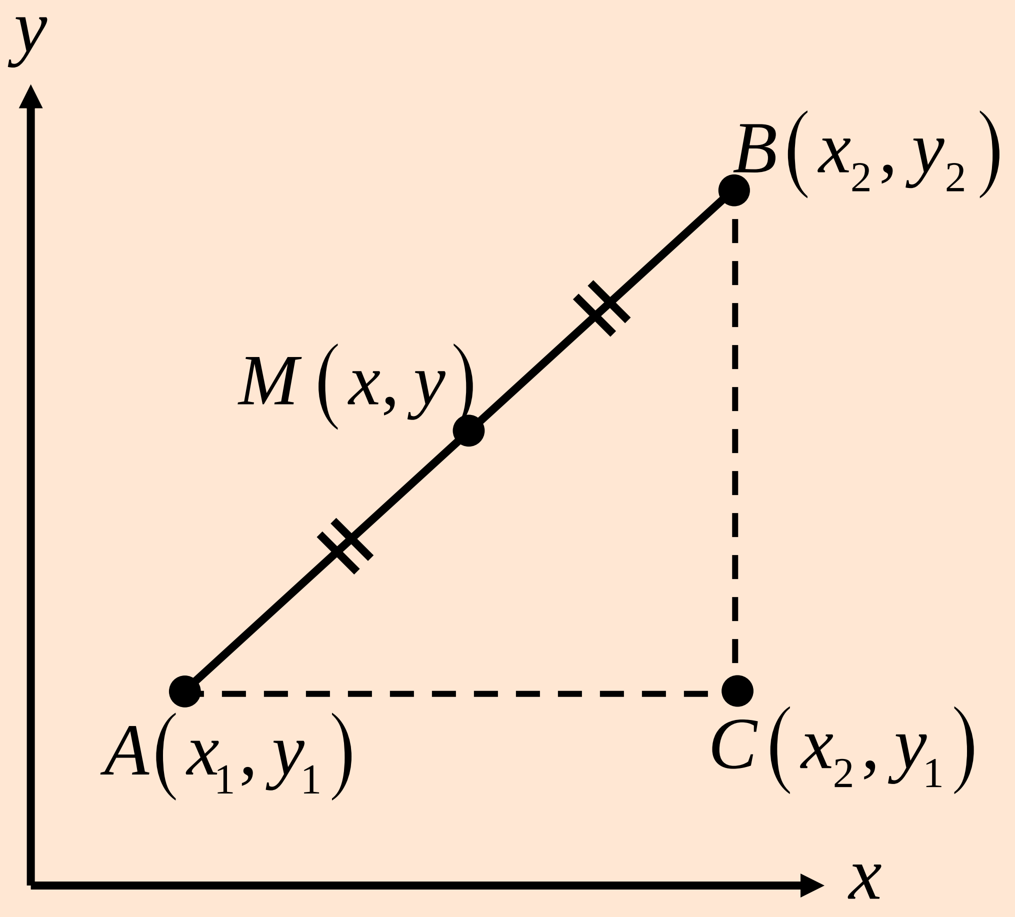


Prior Knowledge

Form 2:
Distance in the Cartesian Coordinate System

Distance between 2 points,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



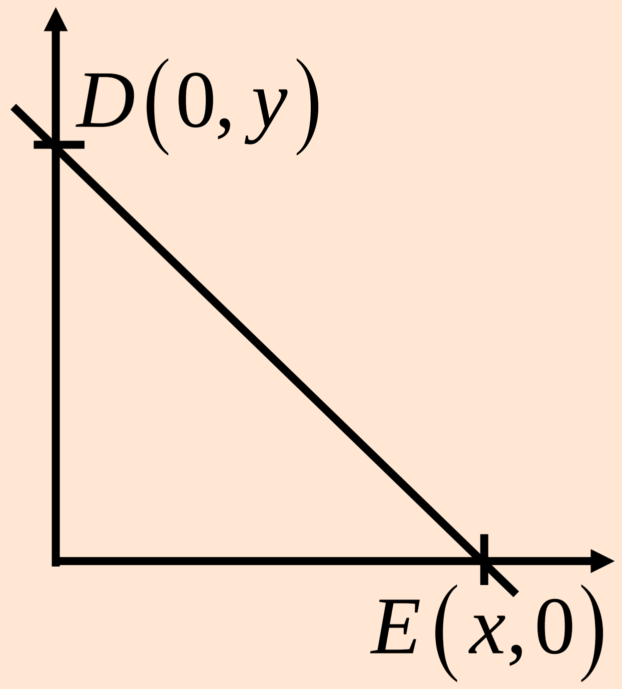
Form 2:
Midpoint in the Cartesian Coordinate System

Midpoint is a point that divides a line segment equally

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Form 2:
Gradient of A Straight Line

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$



$$m_{DE} = -\frac{y - intercept}{x - intercept}$$

Prior Knowledge

3 forms of straight line equation :

(i) Gradient form

$y = mx + c$, where m is gradient and c is y-intercept.

(ii) General form

$ax + by + c = 0$

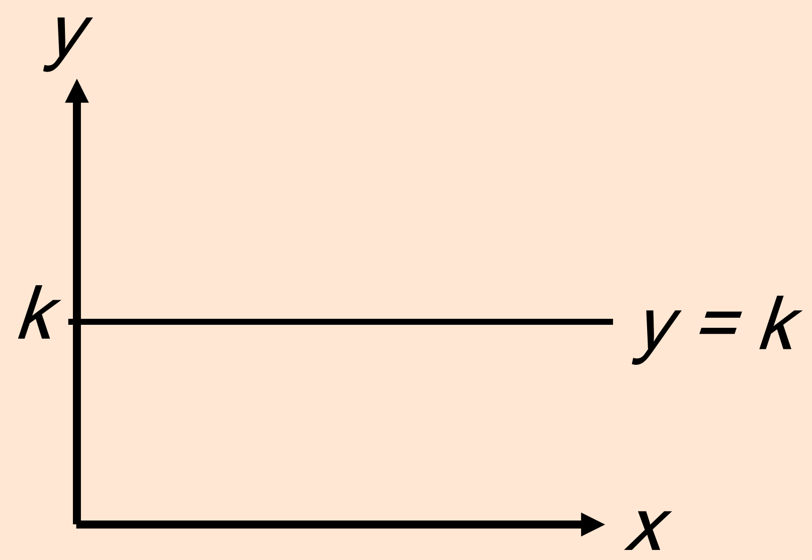
(iii) Intercept form

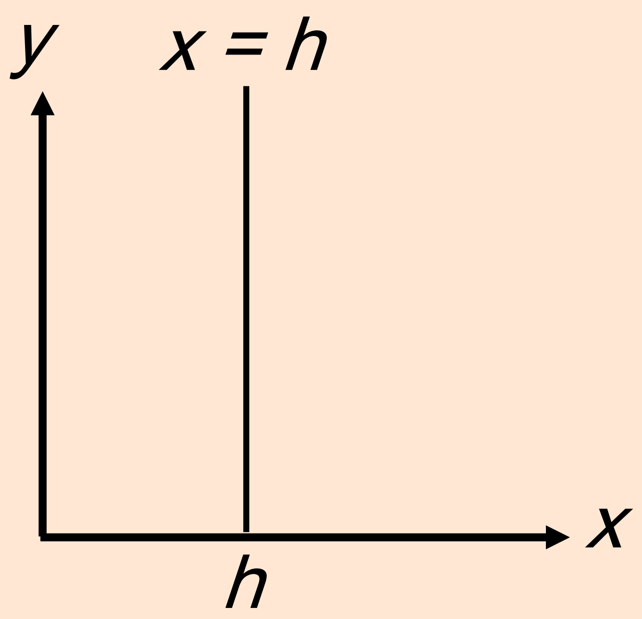
$\frac{x}{a} + \frac{y}{b} = 1$,

a = x-intercept

b = y-intercept

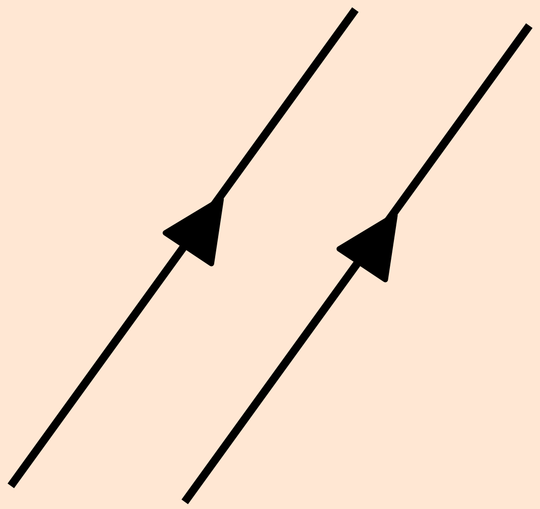
Gradient = $-\frac{b}{a}$.



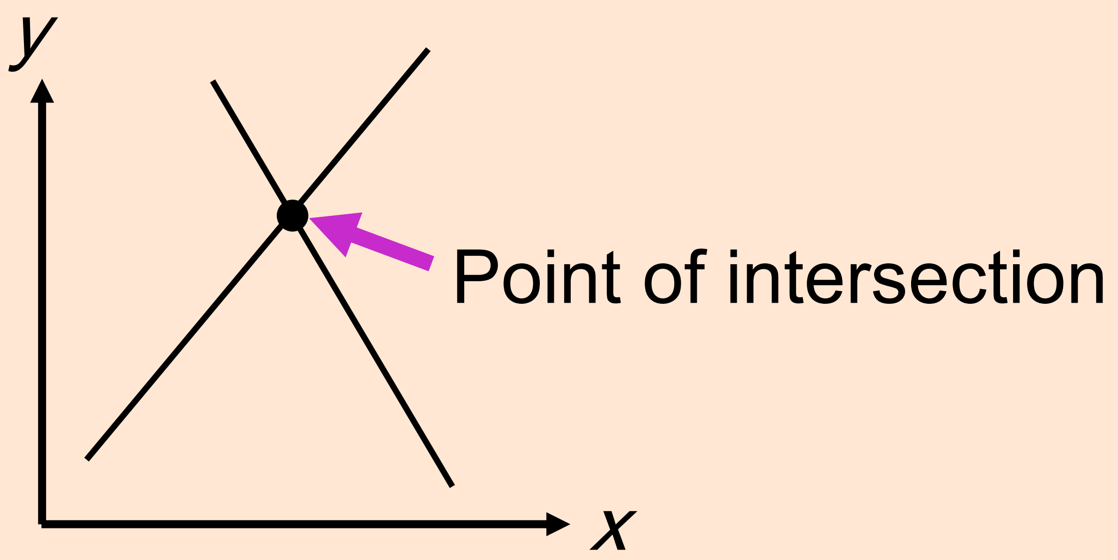


Form 3:

9.0 Straight Lines



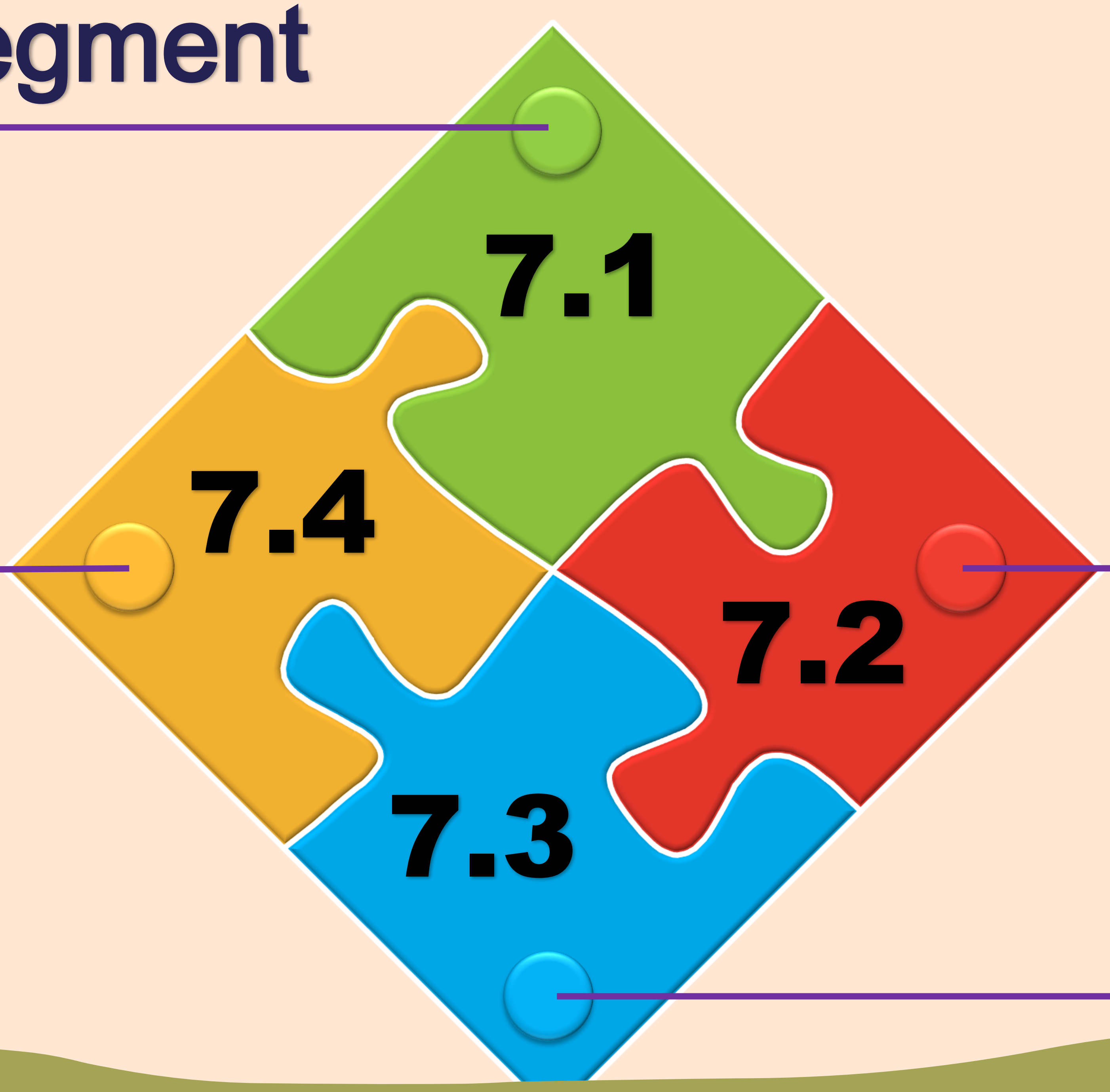
$m_1 = m_2$



7. Coordinates Geometry

Divisor of a line segment

Equations of loci

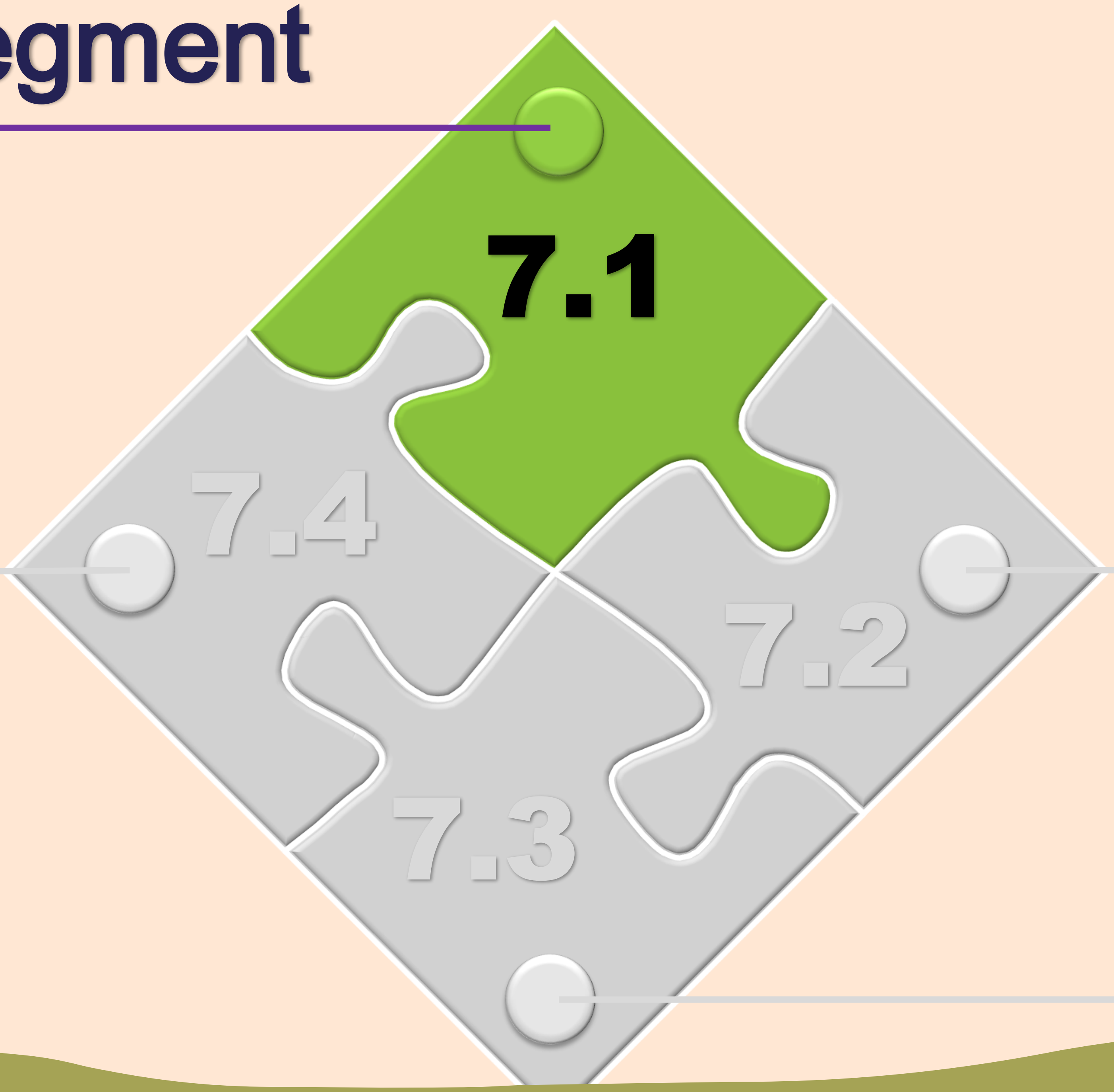


Parallel and perpendicular lines

Area of polygons

7. Coordinates Geometry

Divisor of a line segment

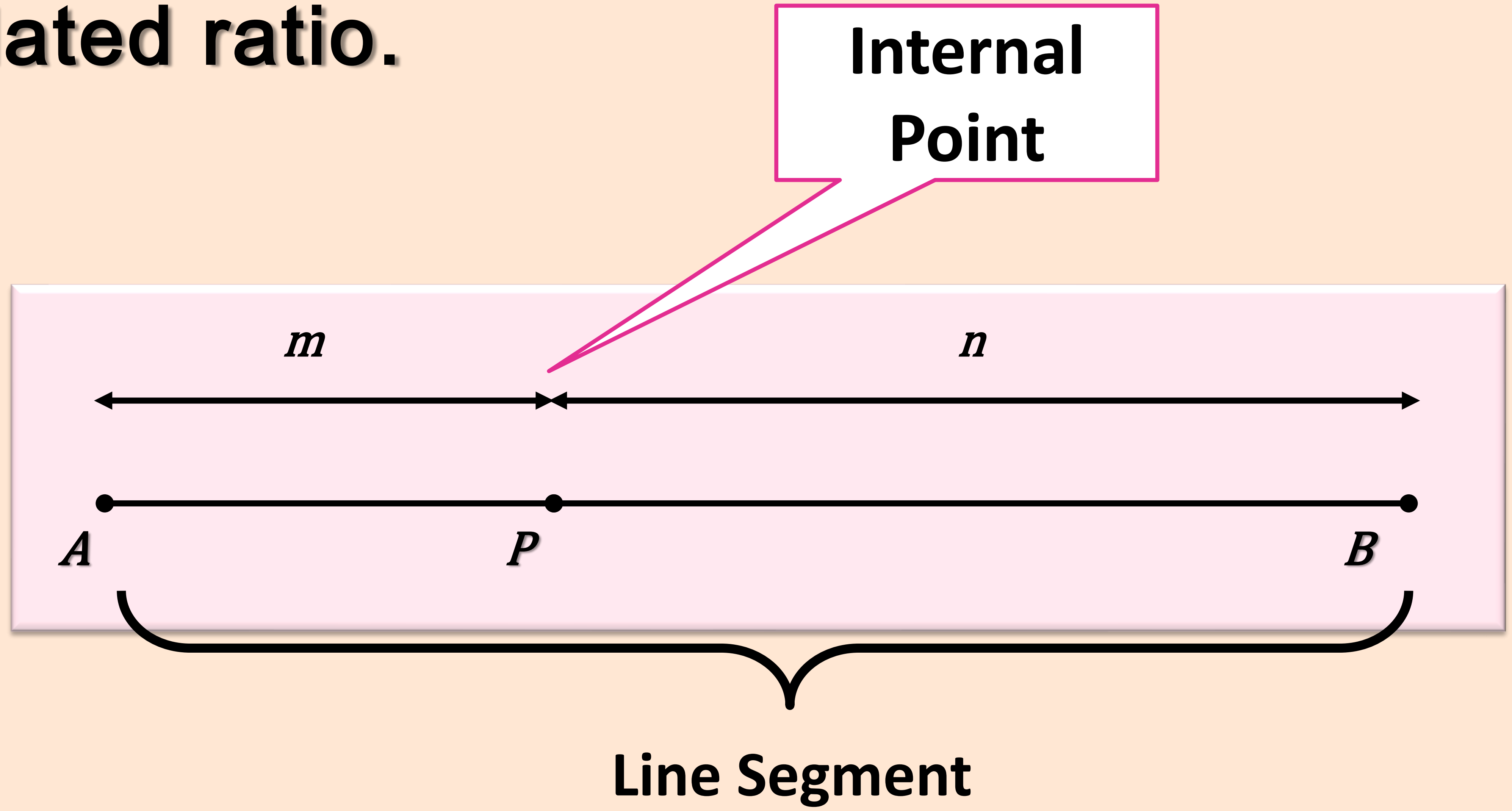


Parallel and perpendicular lines

Area of polygons

Equations of loci

7.1.1 Relate the position of a point that divides a line segment with the related ratio.



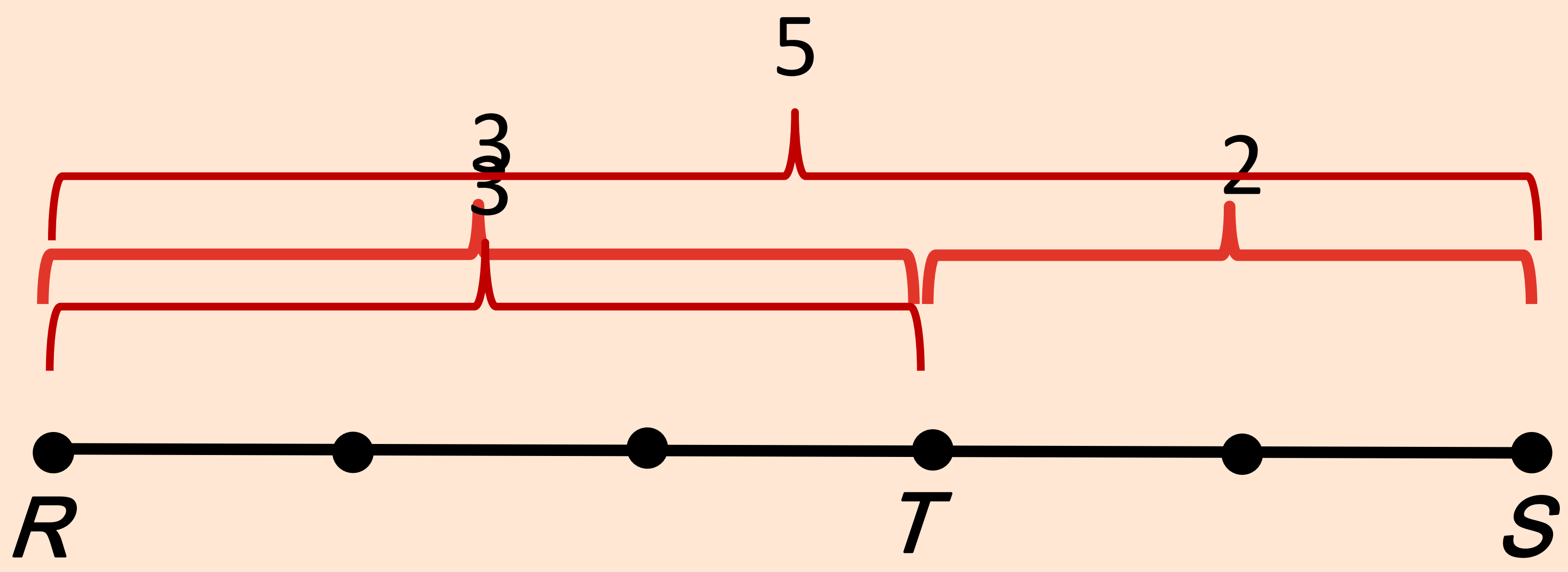
$$AP : PB = m : n$$

If $m = n$, P is the midpoint of line segment AB

https://www.geogebra.org/m/vW2CaktT

Divisor of a Line Segment

Point T is lies on the line segment *RS*, where point *T* is $\frac{3}{5}$ of the distance *RS* from point *R* along the line segment *RS*.

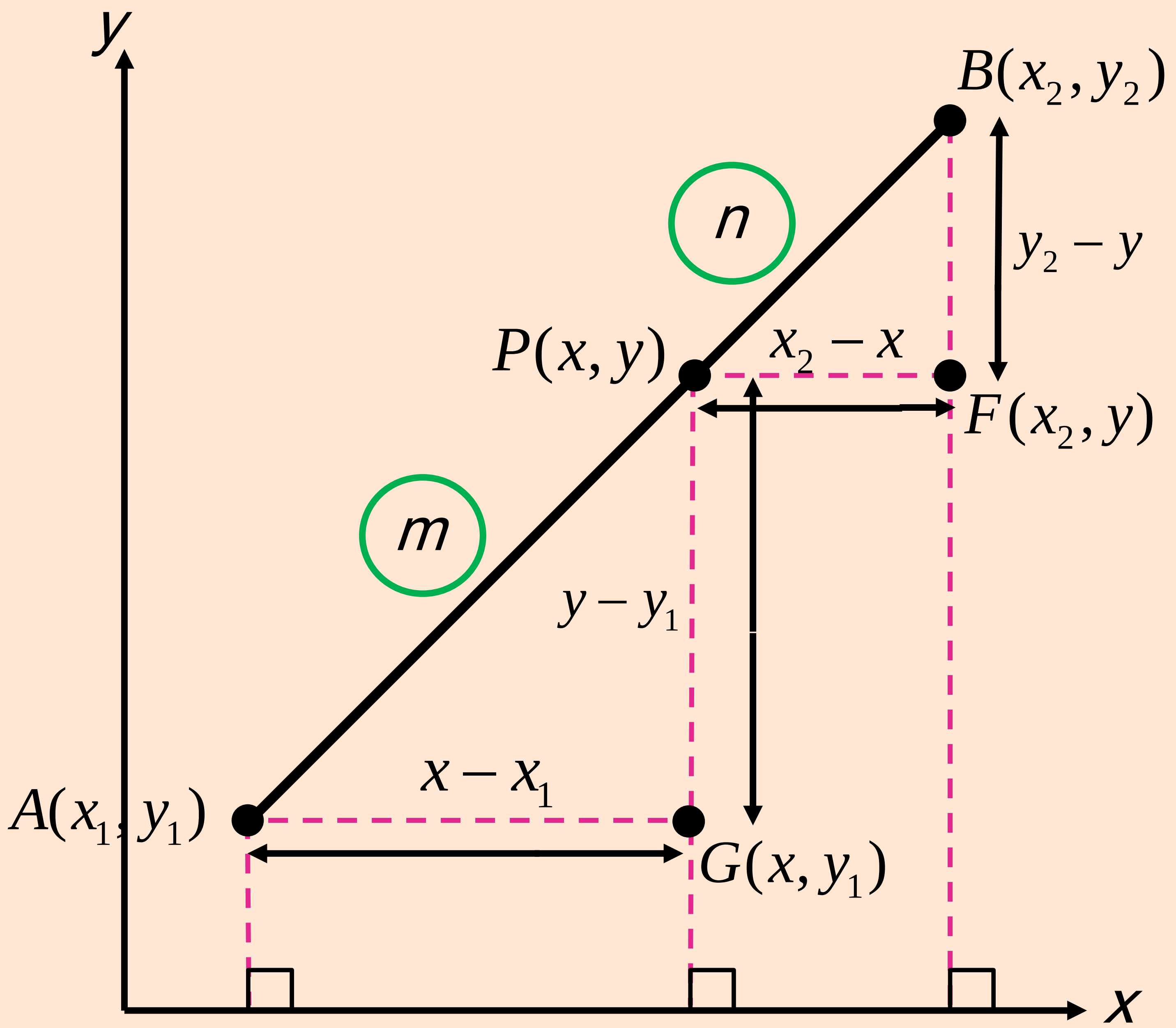


i. $RT : TS = 3 : 2$

ii. $RT : RS = 3 : 5$

7.1.2 Derive the formula for divisor of a line segment on a Cartesian plane.

$P(x, y)$ is a point which divides line segment AB in the ratio $m : n$.



$$\frac{AG}{PF} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x_2 - x} = \frac{m}{n}$$

$$n(x - x_1) = m(x_2 - x)$$

$$nx - nx_1 = mx_2 - mx$$

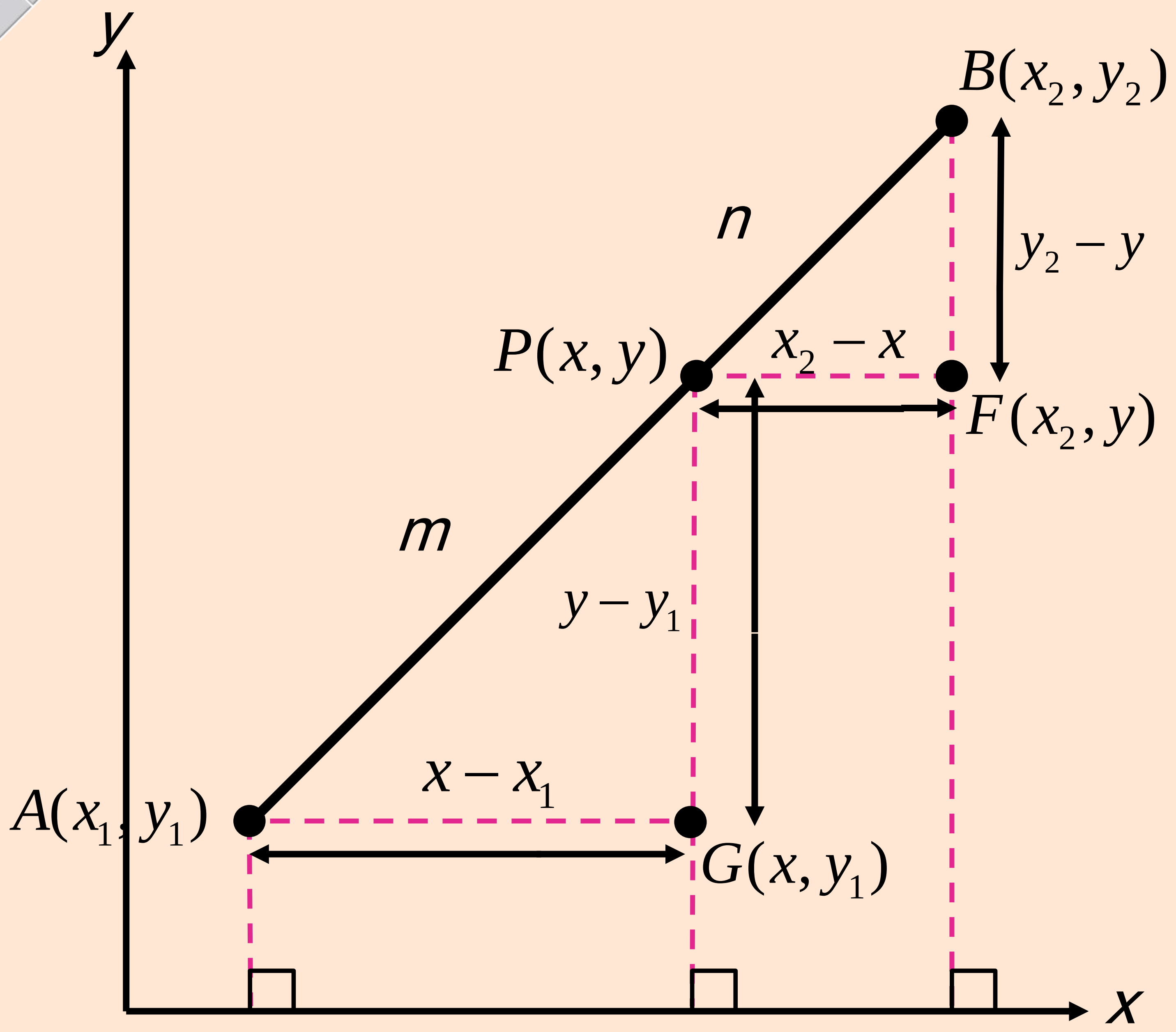
$$mx + nx = nx_1 + mx_2$$

$$x(m + n) = nx_1 + mx_2$$

$$x = \frac{nx_1 + mx_2}{m + n}$$

Divisor of a Line Segment

7.1.2 Derive the formula for divisor of a line segment on a Cartesian plane.



$$\frac{PG}{BF} = \frac{AP}{PB}$$

$$\frac{y - y_1}{y_2 - y} = \frac{m}{n}$$

$$n(y - y_1) = m(y_2 - y)$$

$$ny - ny_1 = my_2 - my$$

$$my + ny = ny_1 + my_2$$

$$y(m + n) = ny_1 + my_2$$

$$y = \frac{ny_1 + my_2}{m + n}$$

Thus, the coordinates of point $P\,(x,y)$ which divides the line segment joining points $A\,(x_1,y_1)$ and $B\,(x_2, y_2)$ in the ratio $m : n$ is :

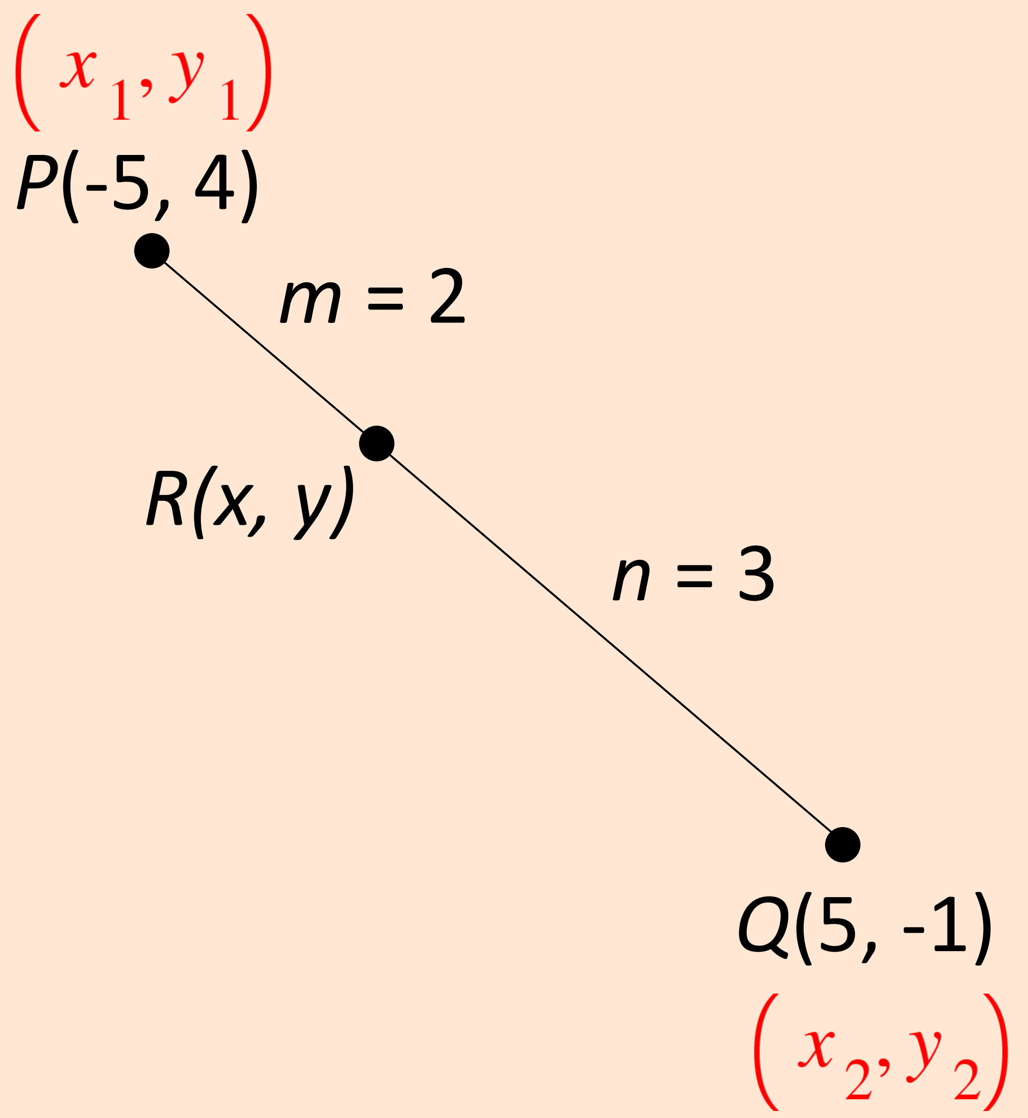
$$P(x,y) = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n}\right)$$

Divisor of a Line Segment

Example 1

The coordinates of points P and Q are (-5,4) and (5,-1) respectively. If point R divides line segment PQ in the ratio 2 : 3, find the coordinates of point R.

Solution:



$$R(x, y) = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n}\right)$$

$$= \left(\frac{3(-5) + 2(5)}{2 + 3}, \frac{3(4) + 2(-1)}{2 + 3}\right)$$

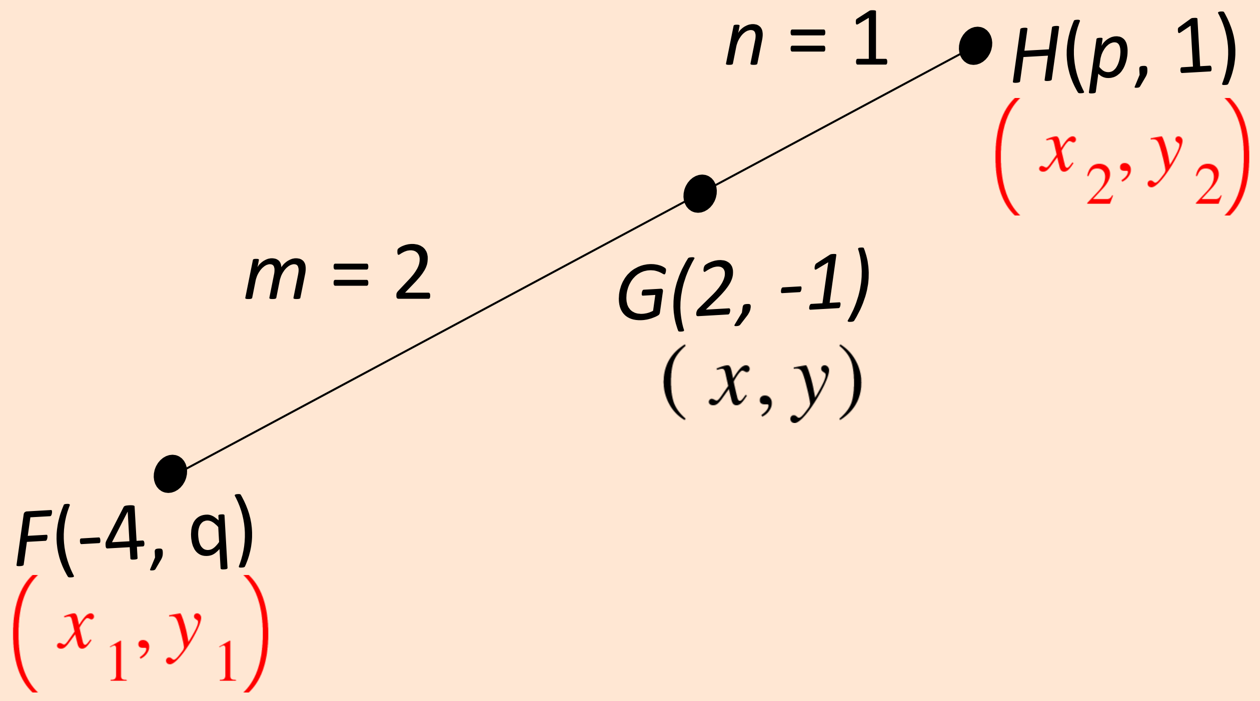
$$= \left(\frac{-5}{5}, \frac{10}{5}\right)$$

$$= (-1 , 2)$$

7.1.3 Solve problems involving divisor of a line segment.

Example 2

It is given that $F(-4, q)$, $G(2, -1)$ and $H(p, 1)$ are collinear such that $FG : GH = 2 : 1$.
Find the values of p and q.



Solution:

$FG : GH = 2 : 1$

$m : n = 2 : 1$

$$(x, y) = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n}\right)$$

$$(2, -1) = \left(\frac{1(-4) + 2(p)}{2 + 1}, \frac{1(q) + 2(1)}{2 + 1}\right)$$

For x-coordinate,

$$\frac{1(-4) + 2(p)}{2 + 1} = 2$$

$$-4 + 2p = 6$$

$$2p = 10$$

$$p = 5$$

For y-coordinate,

$$\frac{1(q) + 2(1)}{2 + 1} = -1$$

$$q + 2 = -3$$

$$q = -5$$

Divisor of a Line Segment

Example 3a

The coordinates of the points *A* and *B* are (3,2) and (*k*,10) respectively. If the point P(6,8) lies on the straight line *AB*, find

a) the ratio *AP* : *PB*,

Solution:

a) Equating the *y*-coordinates,

$$\frac{n(2) + m(10)}{m + n} = 8$$

$$2n + 10m = 8m + 8n$$

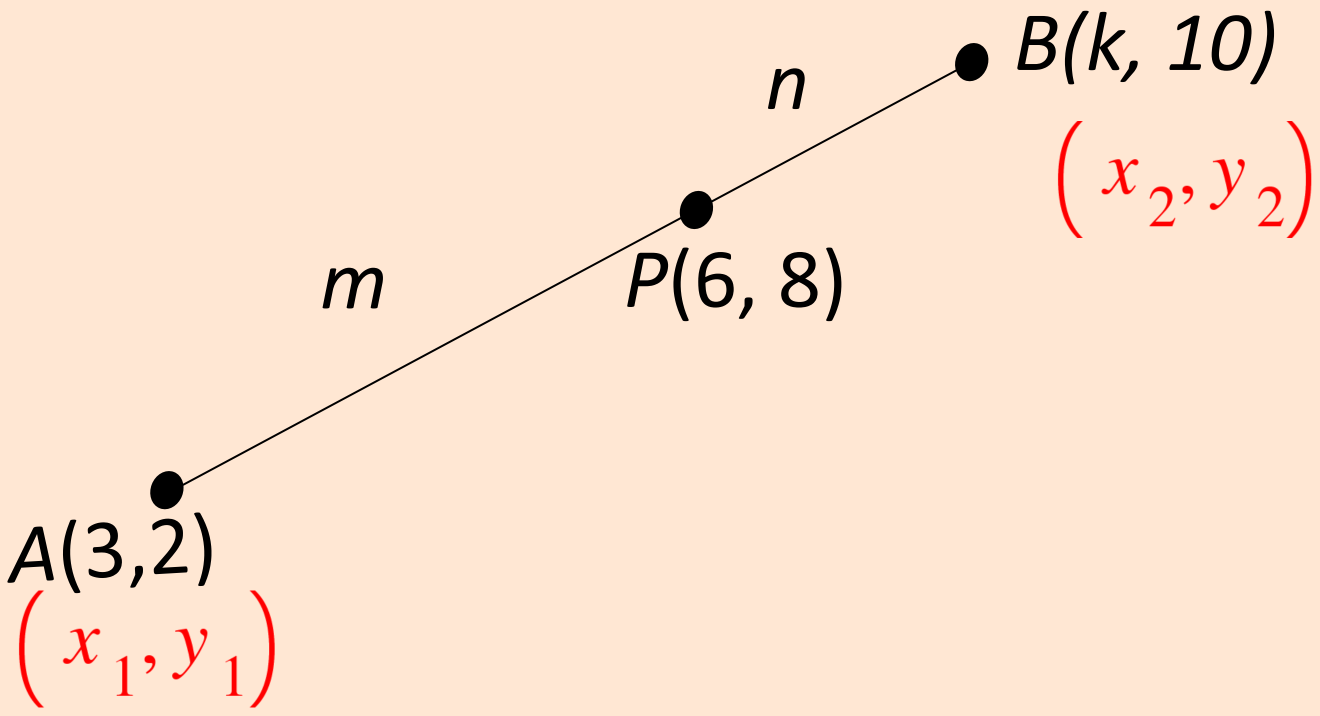
$$2m = 6n$$

$$\frac{m}{n} = \frac{6}{2}$$

$$\frac{m}{n} = \frac{3}{1}$$

$$m : n = 3 : 1$$

$$AP : PB = 3 : 1$$



Example 3b

The coordinates of the points A and B are (3,2) and (k,10) respectively. If the point P(6,8) lies on the straight line AB, find

b) the value of k.

Solution:

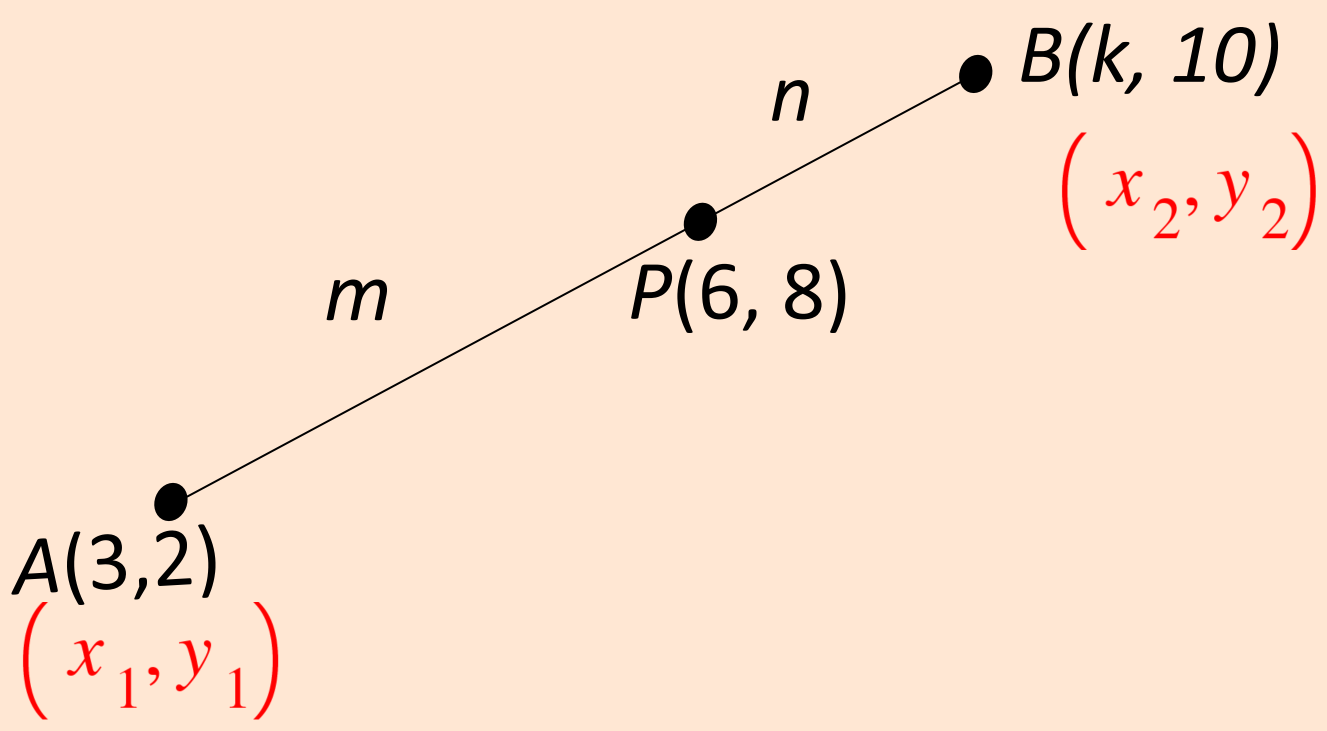
b) Equating the x coordinates,

$$\frac{1(3) + 3(k)}{3 + 1} = 6$$

$$3 + 3k = 24$$

$$3k = 21$$

$$k = 7$$



Kertas 1 SPMRSM 2017

Diagram 1 shows a straight line *PR*.

The point *Q* lies on *PR* such that *PQ* : *QR* = 3 : 2.
Find the coordinates of *P*. [3 marks]

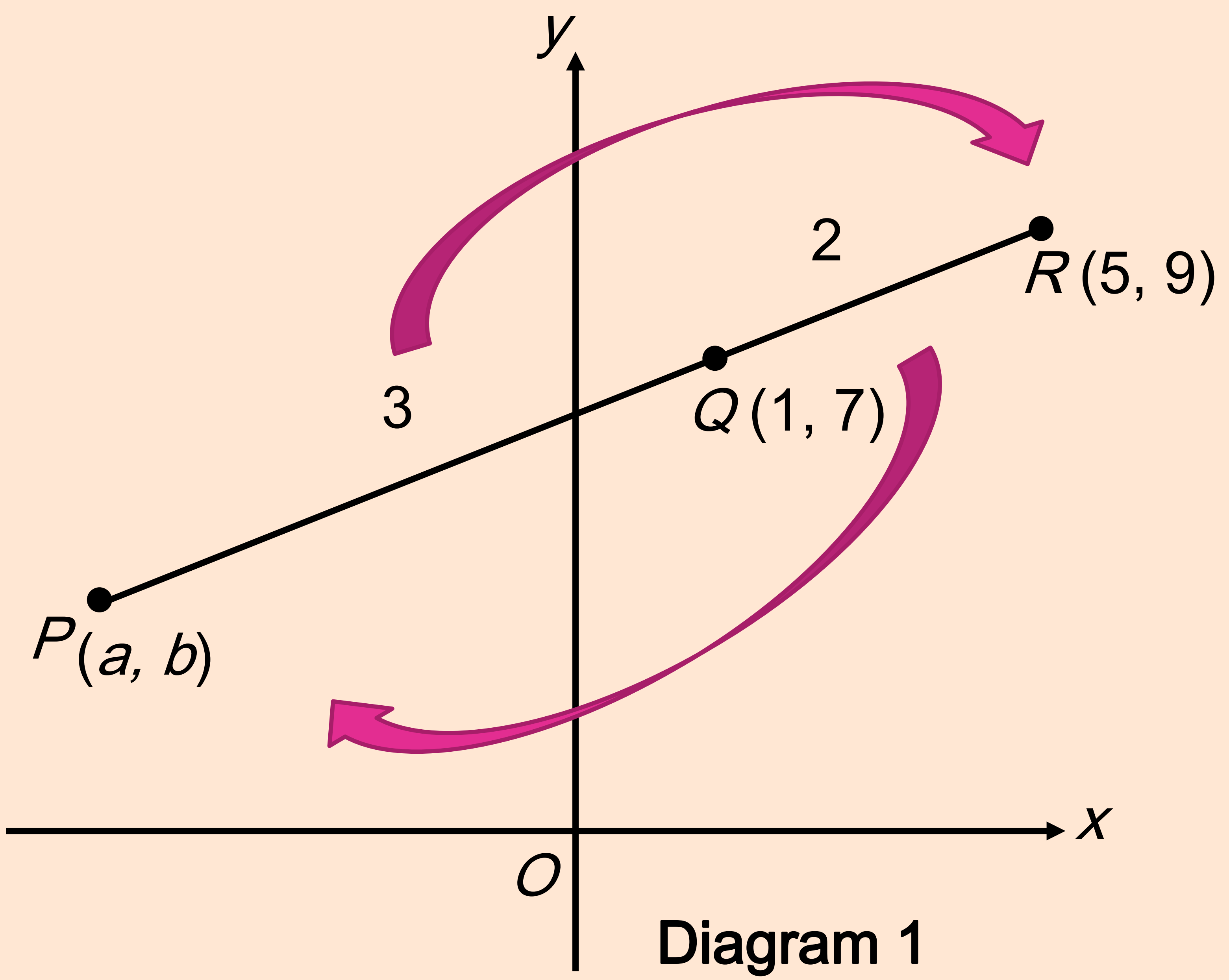


Diagram 1

Solution :-

$$(x, y) = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n}\right)$$

$$(1, 7) = \left(\frac{2a + 3(5)}{3 + 2}, \frac{2b + 3(9)}{3 + 2}\right)$$

$$\frac{2a + 3(5)}{3 + 2} = 1 \qquad \frac{2b + 3(9)}{3 + 2} = 7$$

$2a + 15 = 5$ $2b + 27 = 35$

$a = -5$ $b = 4$

Coordinates of *P* = (-5, 4)

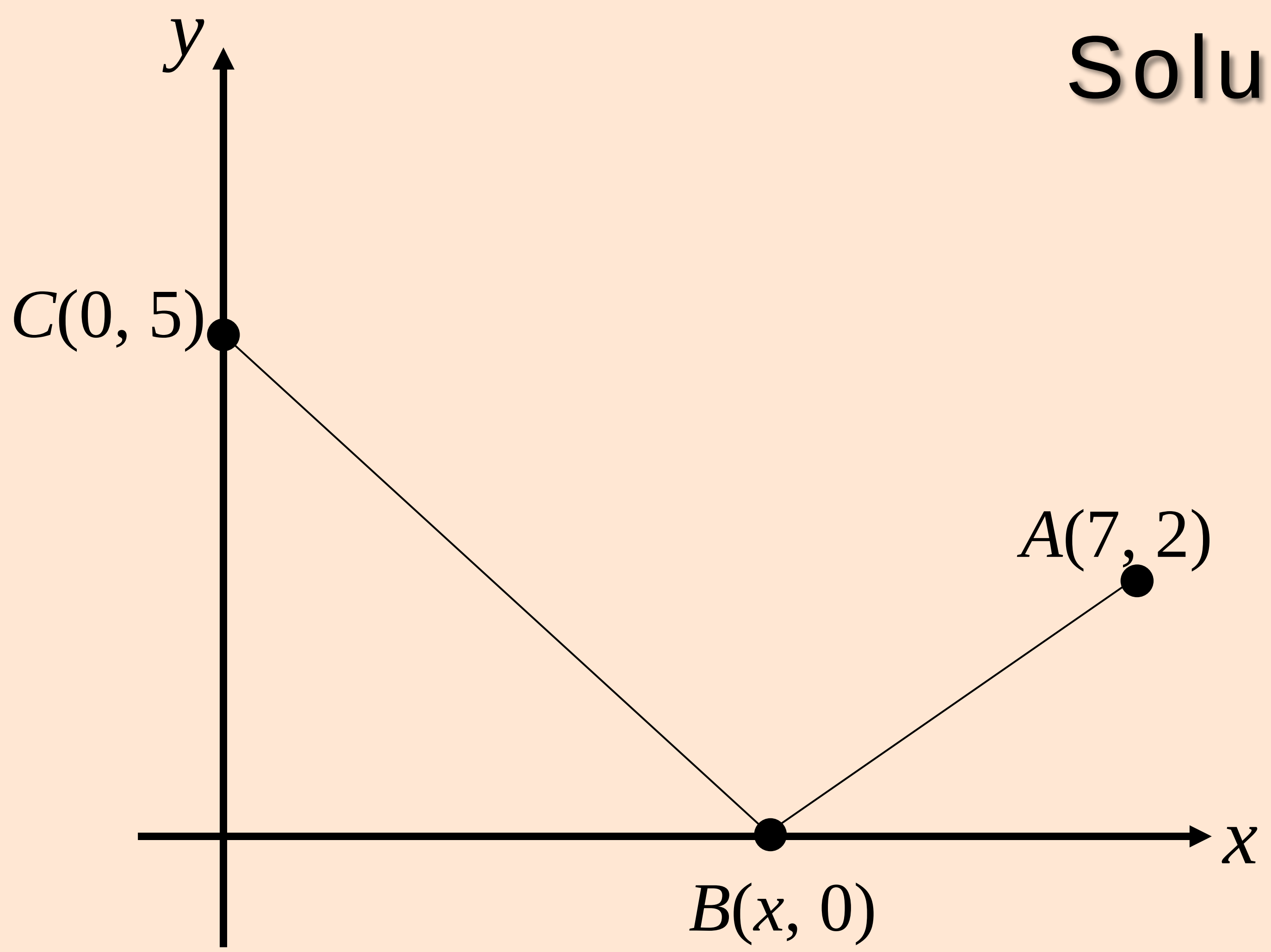
Example 4a

The diagram shows the points $A(7, 2)$, $B(x,0)$ and $C(0,5)$. Given the gradient of straight line AB is $\frac{2}{3}$, find

(a) the equation of AB in the :

(i) gradient form,

(ii) general form.



Solution:- a) (i) $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{2}{3}(x - 7)$$

$$y = \frac{2}{3}x - \frac{14}{3} + 2$$

$$y = \frac{2}{3}x - \frac{8}{3}$$

Gradient form
$y = mx + c$

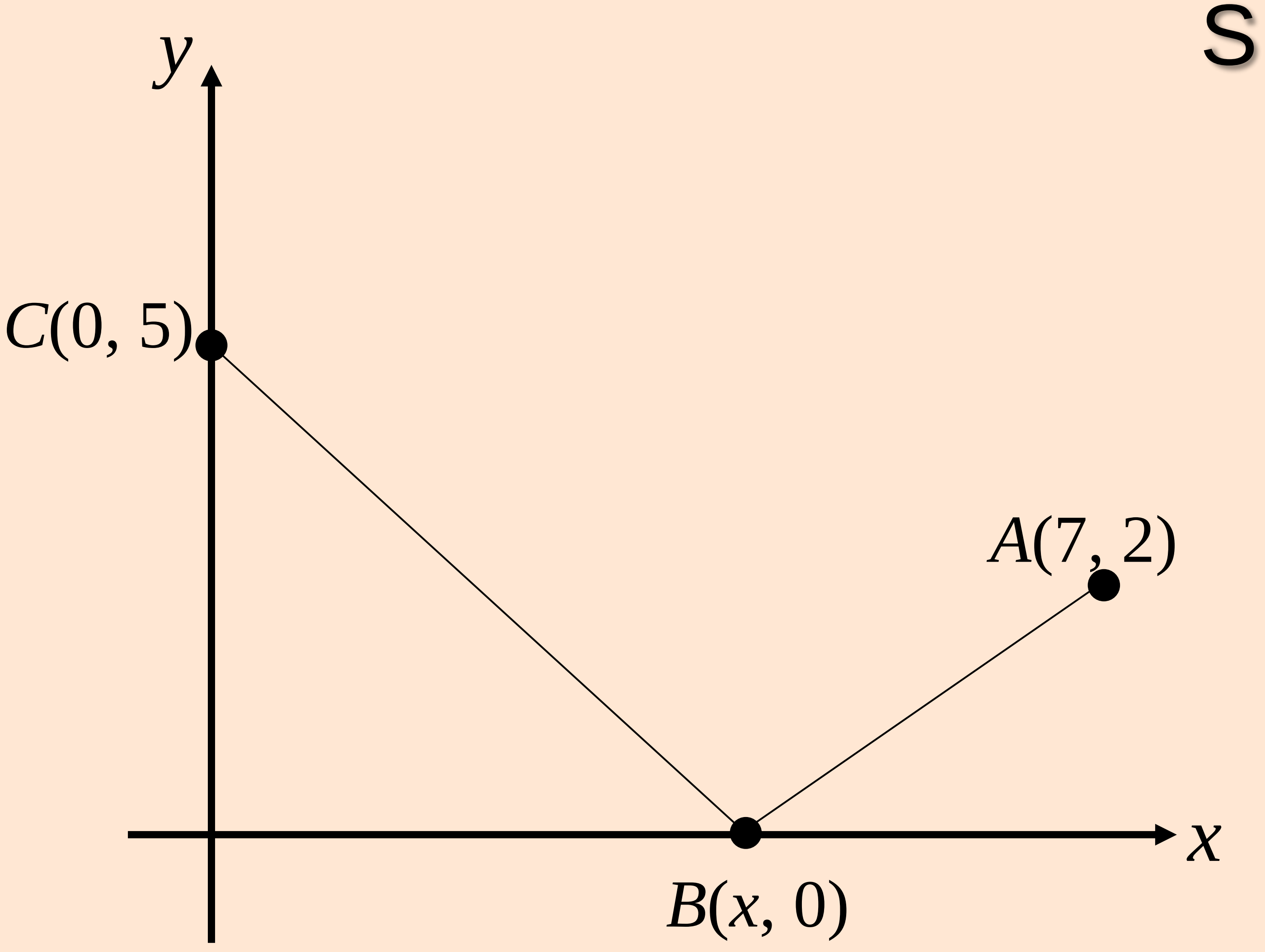
(ii) $2x - 3y - 8 = 0$

General form
ax + by +c = 0

Straight Line

Example 4b

The diagram shows the points $A(7, 2)$, $B(x,0)$ and $C(0,5)$. Find
b) the equation of the straight line BC in the intercept form.



Solution:-

$$2x - 3y - 8 = 0$$

When $y = 0$, $2x = 8$

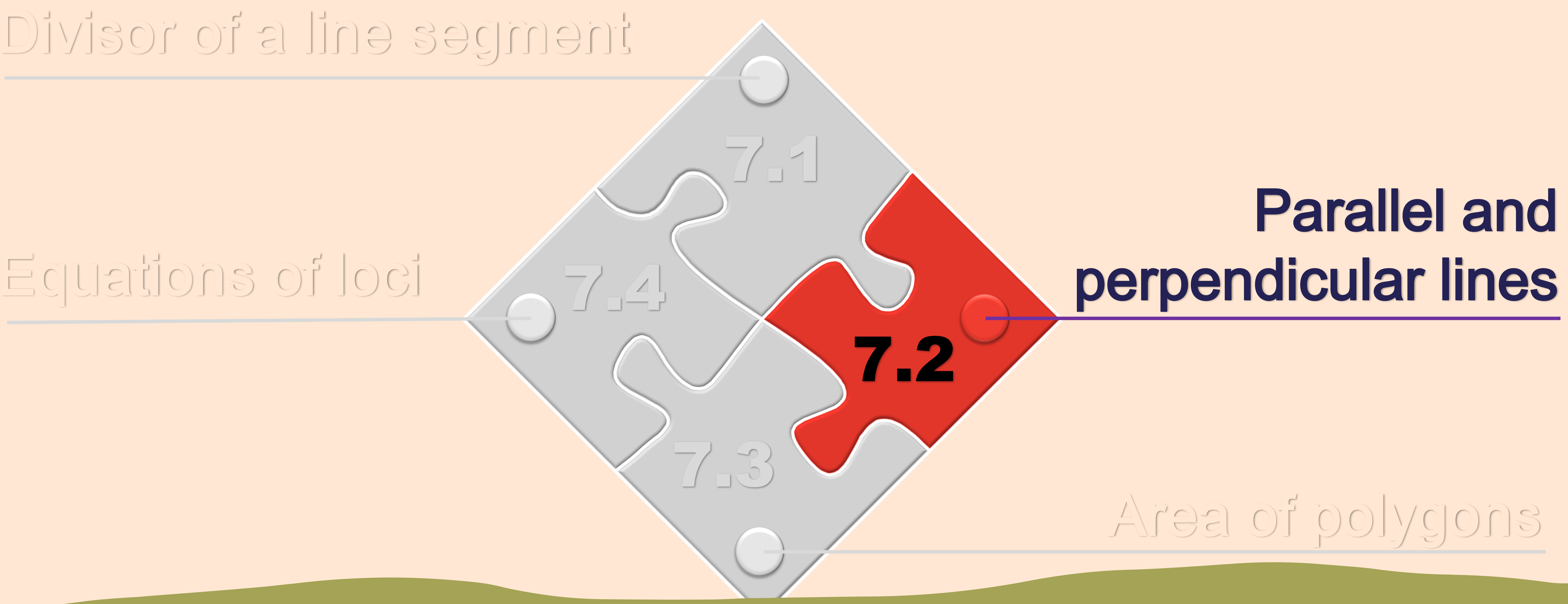
$x = 4$

$$\frac{x}{4} + \frac{y}{5} = 1$$

Intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

7. Coordinates Geometry

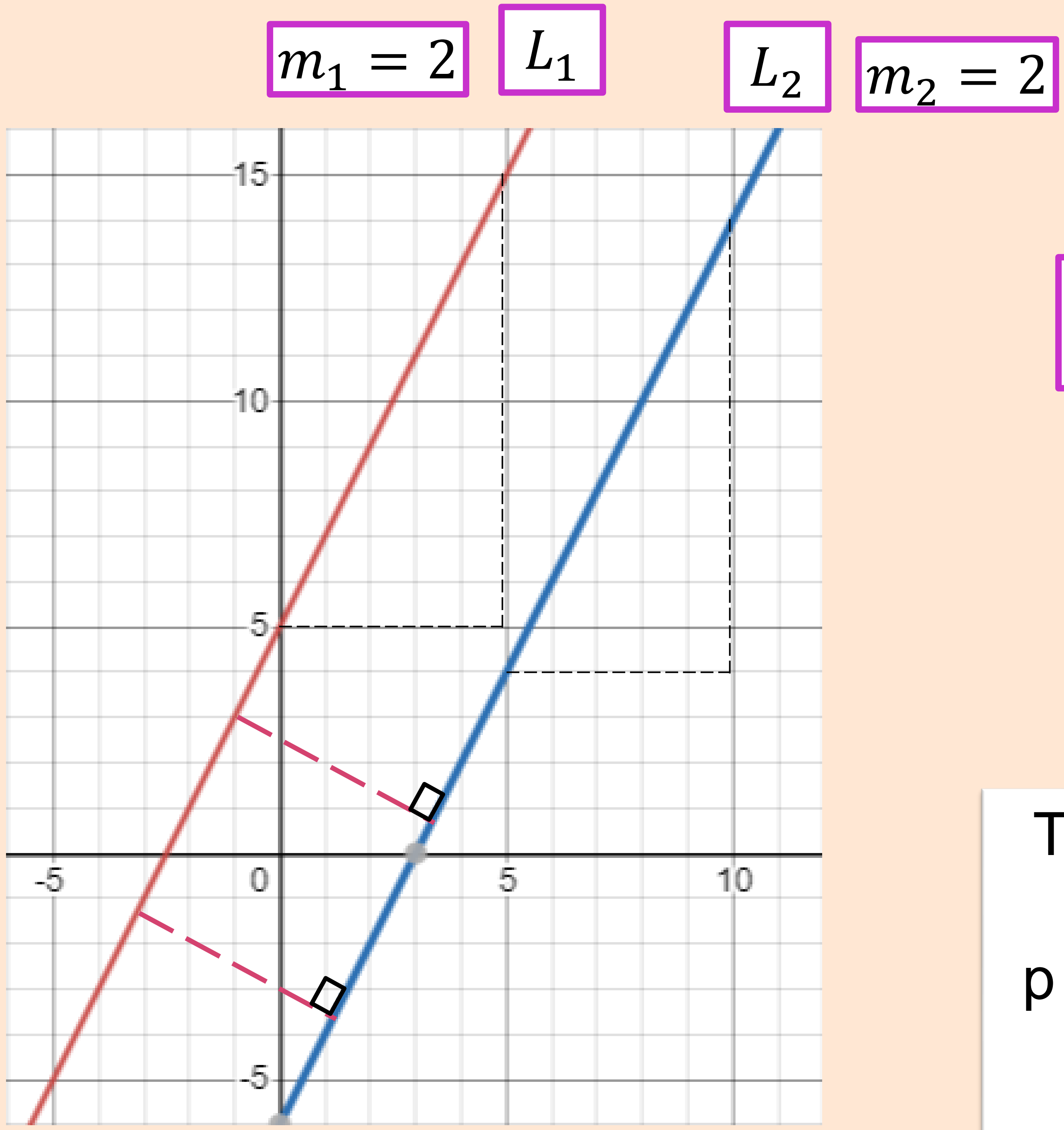


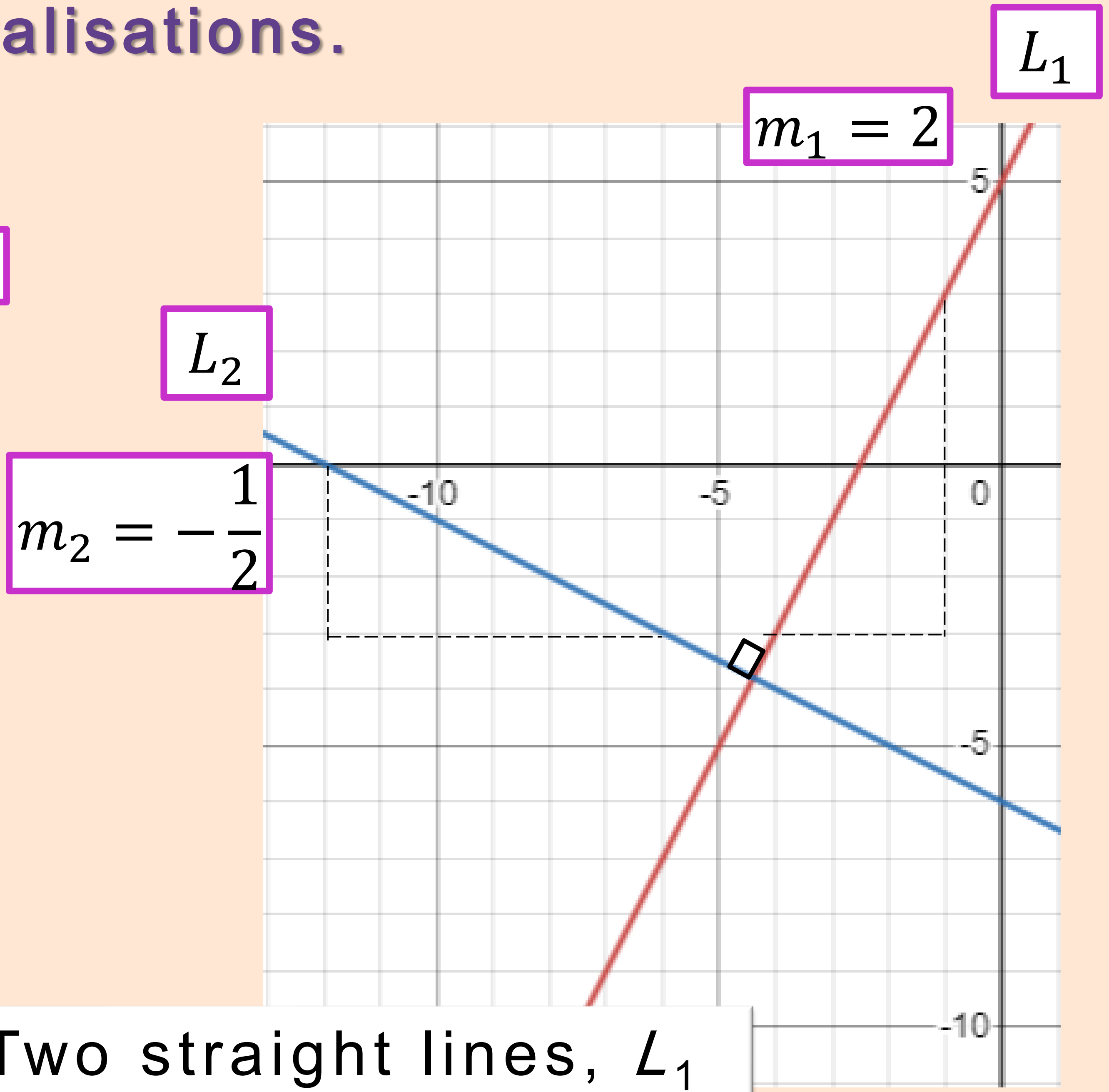
Parallel and perpendicular lines

7.2.1 Make and verify conjectures about gradient of:
(i) parallel lines,
(ii) perpendicular lines and hence, make generalisations.

Two straight lines, L_1 and L_2 are parallel to each other if and only if $m_1 = m_2$

Perpendicular distance between L_1 and L_2 is always the same for parallel cases





Two straight lines, L_1 and L_2 are perpendicular to each other if and only if $m_1 m_2 = -1$.

7.1 7.2 7.3 7.4

Example 5

Determine whether the lines $4x + 6y = 5$ and $2x = 6 - 3y$ are parallel or not.

Solution:-

a) From $6y = -4x + 5$

$$y = \frac{-4x}{6} + \frac{5}{6}$$

$$y = -\frac{2}{3}x + \frac{5}{6}$$

$$m_1 = -\frac{2}{3}$$

From $3y = -2x + 6$

$$y = \frac{-2x}{3} + \frac{6}{3}$$

$$y = -\frac{2}{3}x + 2$$

$$m_2 = -\frac{2}{3}$$

$m_1 = m_2$, the pair of straight lines are parallel.

7.1 7.4 7.2 7.3

Example 6

It is given straight lines $px + 6y - 12 = 0$ and $2x - 3y + 14 = 0$ are parallel. Find the value of p.

Solution:-

From $6y = -px + 12$

$$y = -\frac{p}{6}x + 2$$

$$m_1 = -\frac{p}{6}$$

From $3y = 2x + 14$

$$y = \frac{2}{3}x + \frac{14}{3}$$

$$m_2 = \frac{2}{3}$$

$$m_1 = m_2$$

$$-\frac{p}{6} = \frac{2}{3}$$

$$-p = \frac{2}{3} \times 6$$

$$p = -4$$

Example 7

Determine whether the lines 3y – x = 12 and 3x – 2y = 8 are perpendicular or not.

Solution:-

From $3y = x + 12$

$$y = \frac{1}{3}x + 4$$

$$m_1 = \frac{1}{3}$$

From $2y = 3x - 8$

$$y = \frac{3}{2}x - 4$$

$$m_2 = \frac{3}{2}$$

$$m_1 \, m_2 = \left(\frac{1}{3}\right)\left(\frac{3}{2}\right)$$

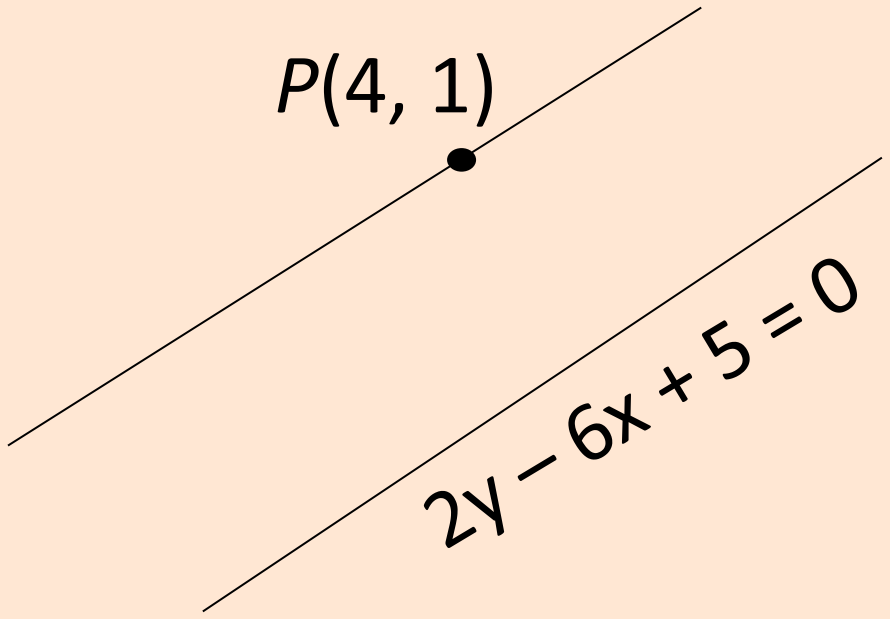
$$= \frac{1}{2} \neq -1$$

The pair of straight lines are not perpendicular

Example 8

Find the equation of the straight line that passes through point P (4, 1) and parallel to the straight line, $2y - 6x + 5 = 0$ in general form.

Solution:-



$2y - 6x + 5 = 0$, $P(4,1)$

$$2y - 6x + 5 = 0$$
$$2y = 6x - 5$$
$$y = 3x - \frac{5}{2}$$
$$m = 3$$

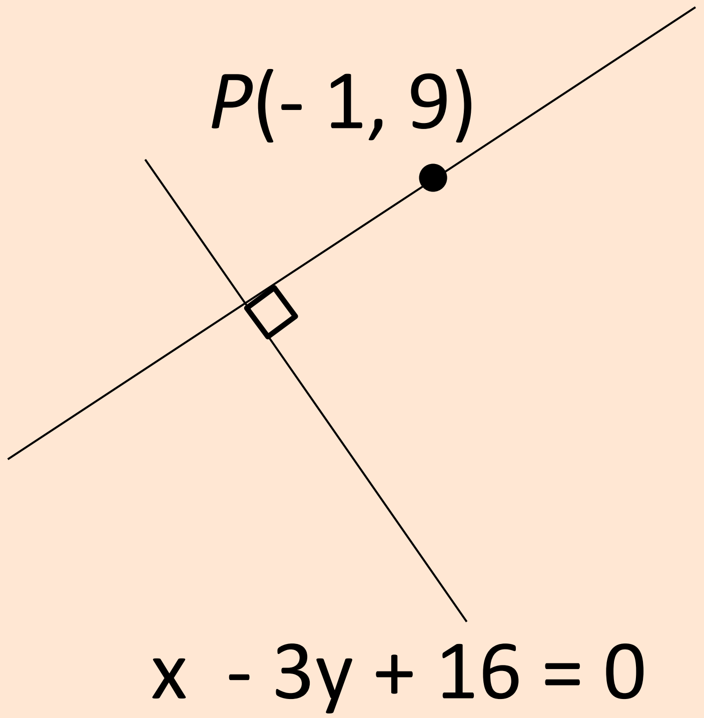
The equation of the straight line:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 3(x - 4)$$
$$y - 1 = 3x - 12$$
$$y = 3x - 11$$
$$3x - y - 11 = 0$$

Example 9

Find the equation of the straight line that passes through point $P(-1,9)$ and perpendicular to the straight line, $x - 3y + 16 = 0$ in intercept form.

Solution:-



$$x - 3y + 16 = 0$$

$$y = \frac{1}{3}x + \frac{16}{3}$$

$$m_1 = \frac{1}{3}$$

$$\frac{1}{3} \times m_2 = -1$$

$$m_2 = -3$$

The equation of the straight line:

$$y - y_1 = m_2(x - x_1)$$

$$y - 9 = -3(x - (-1))$$

$$3x + y = -3 + 9$$

$$\frac{3x}{6} + \frac{y}{6} = \frac{6}{6}$$

$$\frac{x}{2} + \frac{y}{6} = 1$$

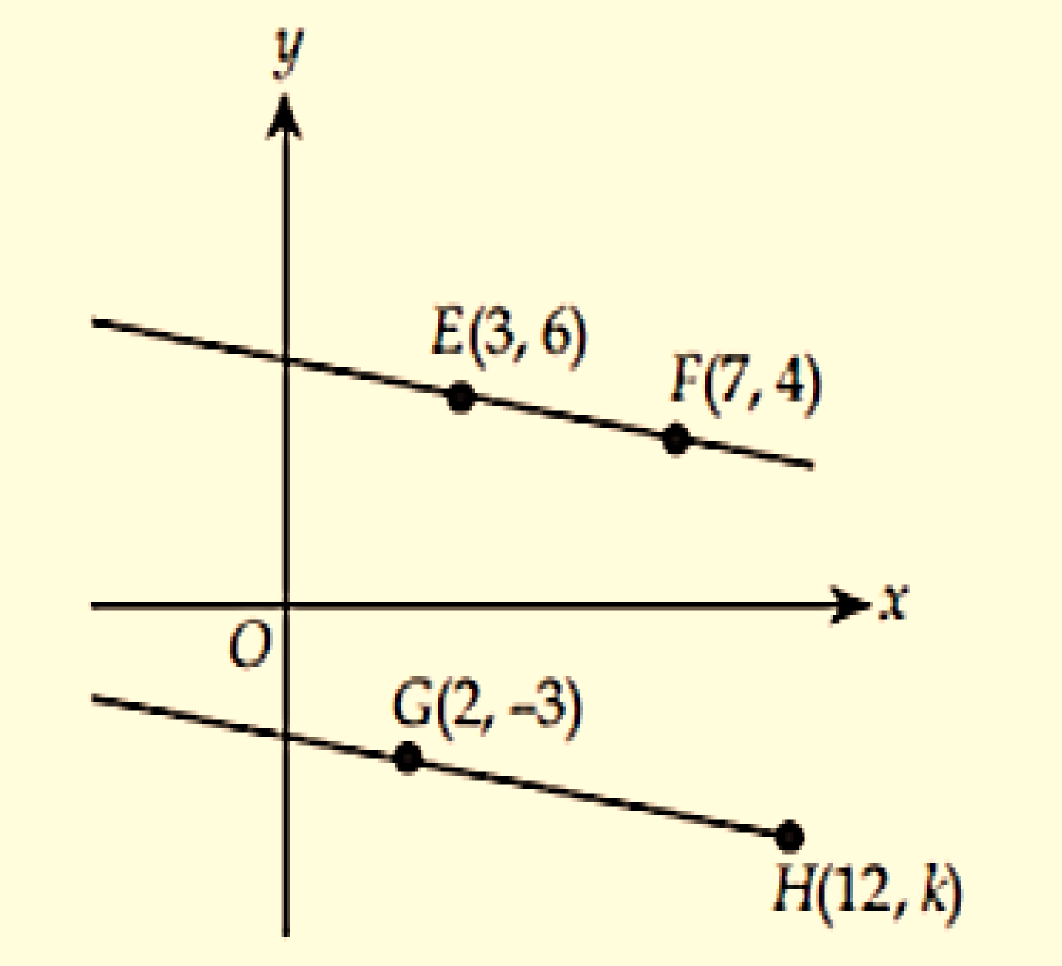
7.2.2 Solve problems involving equations of parallel and perpendicular lines.

Example 10

The diagram below shows the positions of four rest huts, *E, F, G* and *H* in a park. It is given that *EF* is parallel to *GH*. Find

a) the equation of the straight line *GH*,

b) the value of *k*



Solution:-

a) $m_{EF} = \frac{4-6}{7-3}$

$= -\frac{1}{2}$

$m_{GH} = m_{EF}$

$= -\frac{1}{2}$

$y - (-3) = \left(-\frac{1}{2}\right)(x-2)$

$y + 3 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x - 2$

b) When $x = 12$,

$y = k$

$k = -\frac{1}{2}(12) - 2$

$= -8$

Example 11

A developer is required to build a new straight road which passes through the point (–2,5). It is given that a straight road that was already built can be expressed as $2y = 3x - 4$. The developer is required to build the new road such that it does not intersect with the existing road. Determine the equation of the new road.

Solution:-

Gradient of the existing road

$$2y = 3x - 4$$

$$y = \frac{3}{2}x - 2$$

So, $m_1 = \frac{3}{2}$

The new road must be parallel to the existing road, so that both roads do not intersect.

So, $m_2 = \frac{3}{2}$

The equation of the new road,

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{3}{2}(x - (-2))$$

$$y = \frac{3}{2}x + 8$$

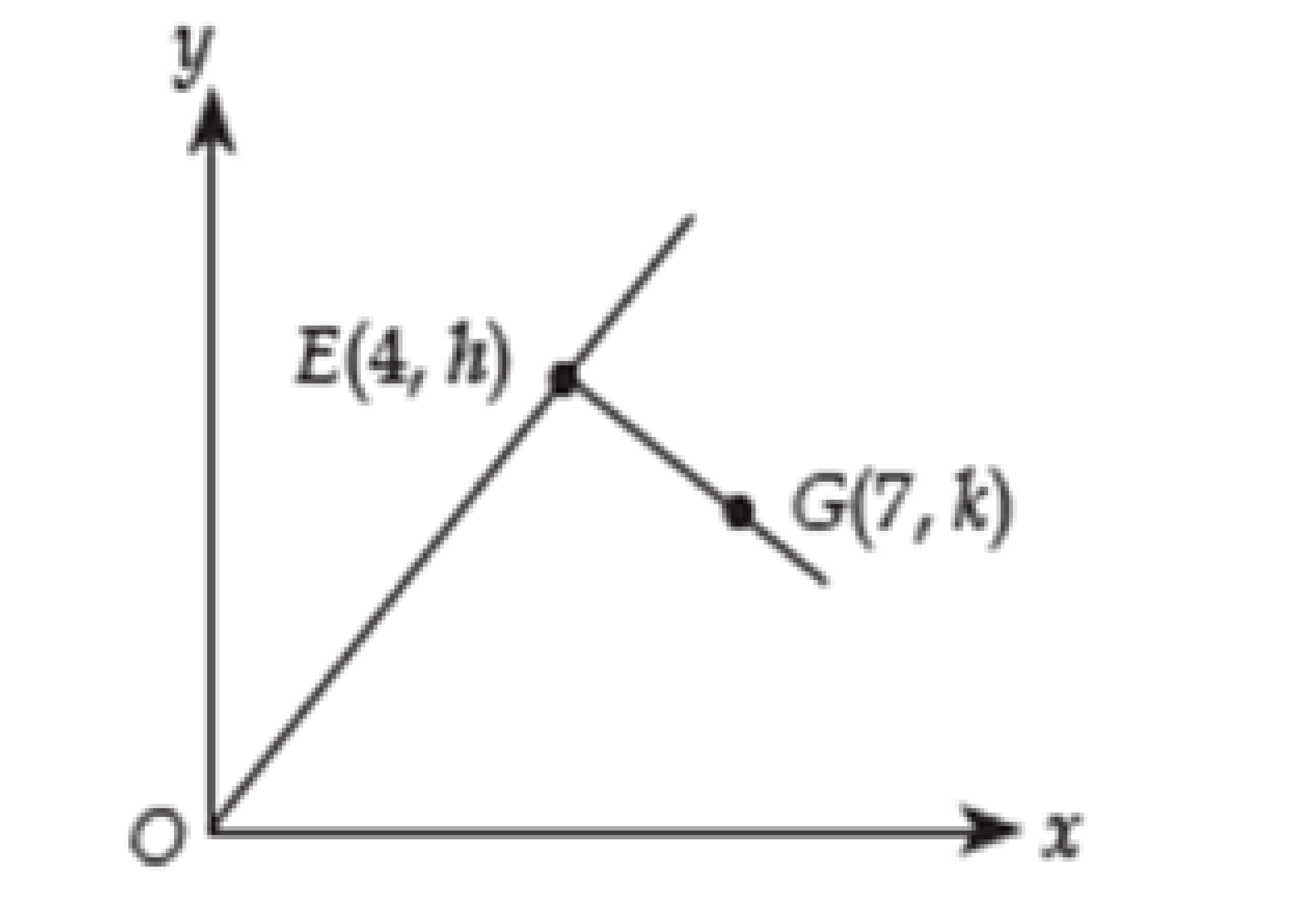
or

$$2y = 3x + 16$$

7.1 7.4 7.2 7.3

Example 12

In the diagram below, E is the point of intersection of two perpendicular lines. If the equation of straight line OE is $y = 3x$, find the values of h and k.



Solution:-

$$y = 3x$$

When $x = 4$, $y = h$

$$h = 3(4)$$

$$h = 12$$

$$m_{OE} = 3$$

$$m_{EG} = -\frac{1}{3}$$

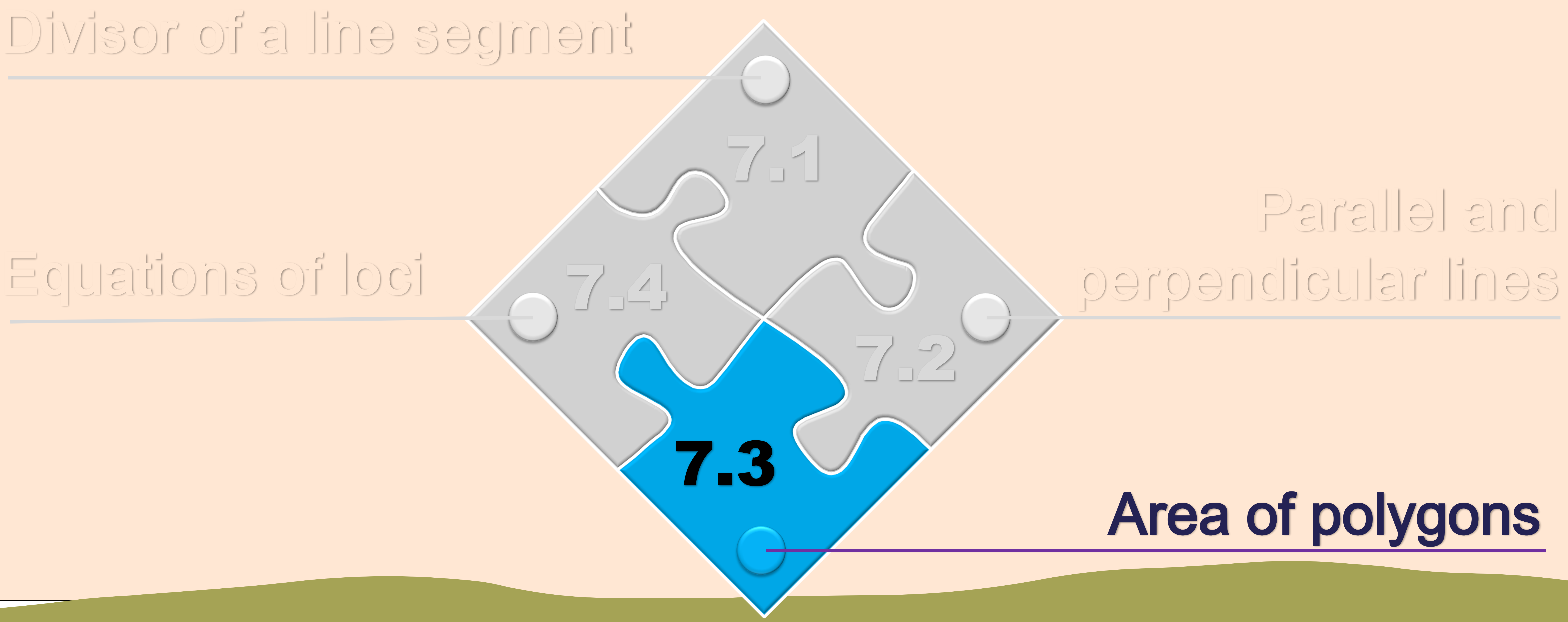
$$\frac{k - h}{7 - 4} = -\frac{1}{3}$$

$$\frac{k - 12}{7 - 4} = -\frac{1}{3}$$

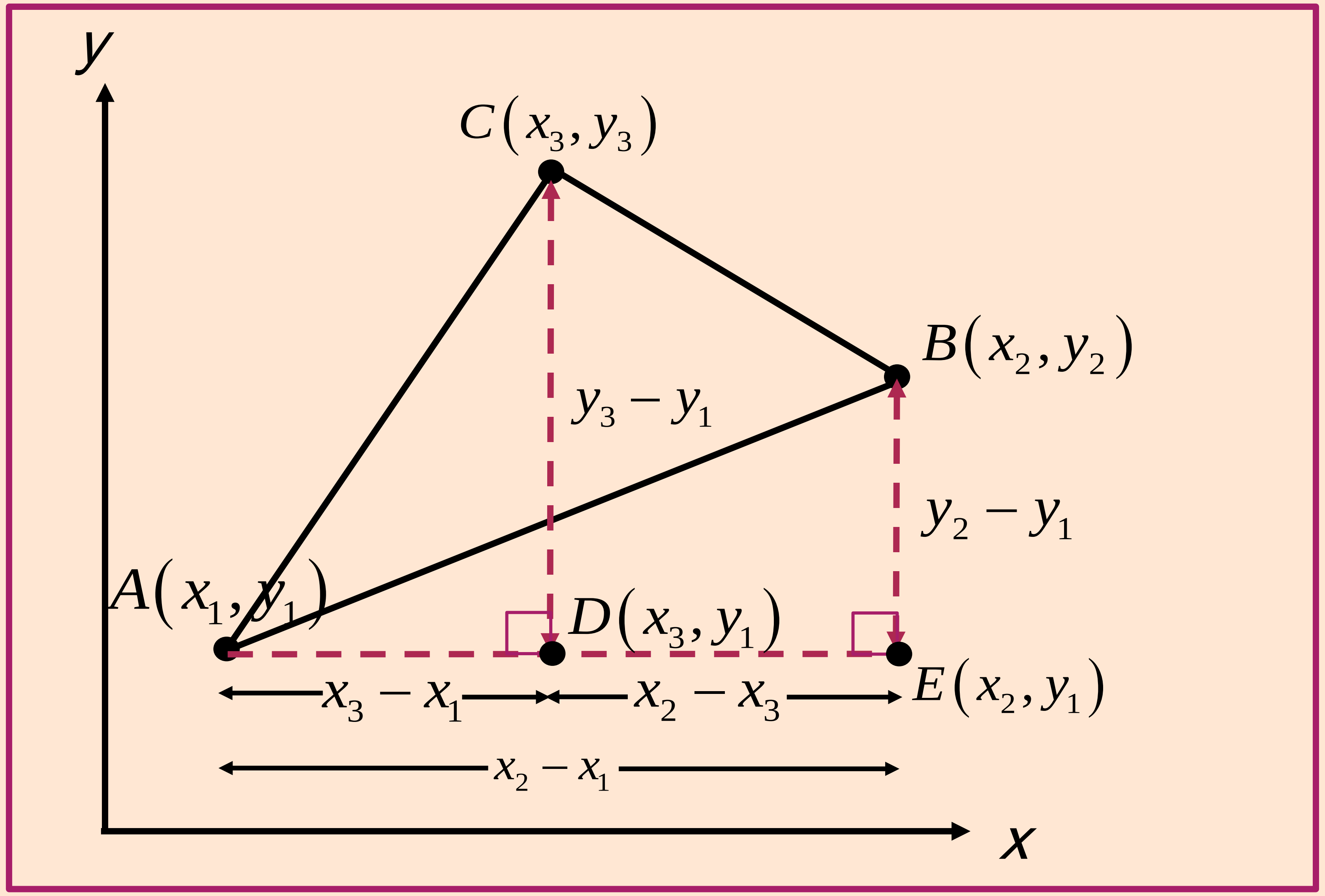
$$k - 12 = -1$$

$$k = 11$$

7. Coordinates Geometry



7.3.1 Derive the formula of area of triangles when the coordinates of each vertex are known.



ABC is a triangle on Cartesian plane

Area of $\triangle ABC$

$$= A_{ACD} + A_{BCDE} - A_{ABE}$$

$$= \frac{1}{2}(AD \times CD) + \frac{1}{2}(DE)(CD + BE) - \frac{1}{2}(AE \times BE)$$

$$= \frac{1}{2}\left[(x_3 - x_1)(y_3 - y_1) + (x_2 - x_3)\left((y_3 - y_1) + (y_2 - y_1)\right) - (x_2 - x_1)(y_2 - y_1)\right]$$

$$= \frac{1}{2}\left(\cancel{x_3y_3} - \cancel{x_3y_1} - x_1y_3 + \cancel{x_1y_1} + x_2y_3 - x_2y_1 + \cancel{x_2y_2} - \cancel{x_2y_1} - \cancel{x_3y_3} + \cancel{x_3y_1} - x_3y_2 + x_3y_1 - \cancel{x_2y_2} + \cancel{x_2y_1} + x_1y_2 - \cancel{x_1y_1}\right)$$

$$= \frac{1}{2}\left(x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3\right)$$

7.3.1 Derive the formula of area of triangles when the coordinates of each vertex are known.

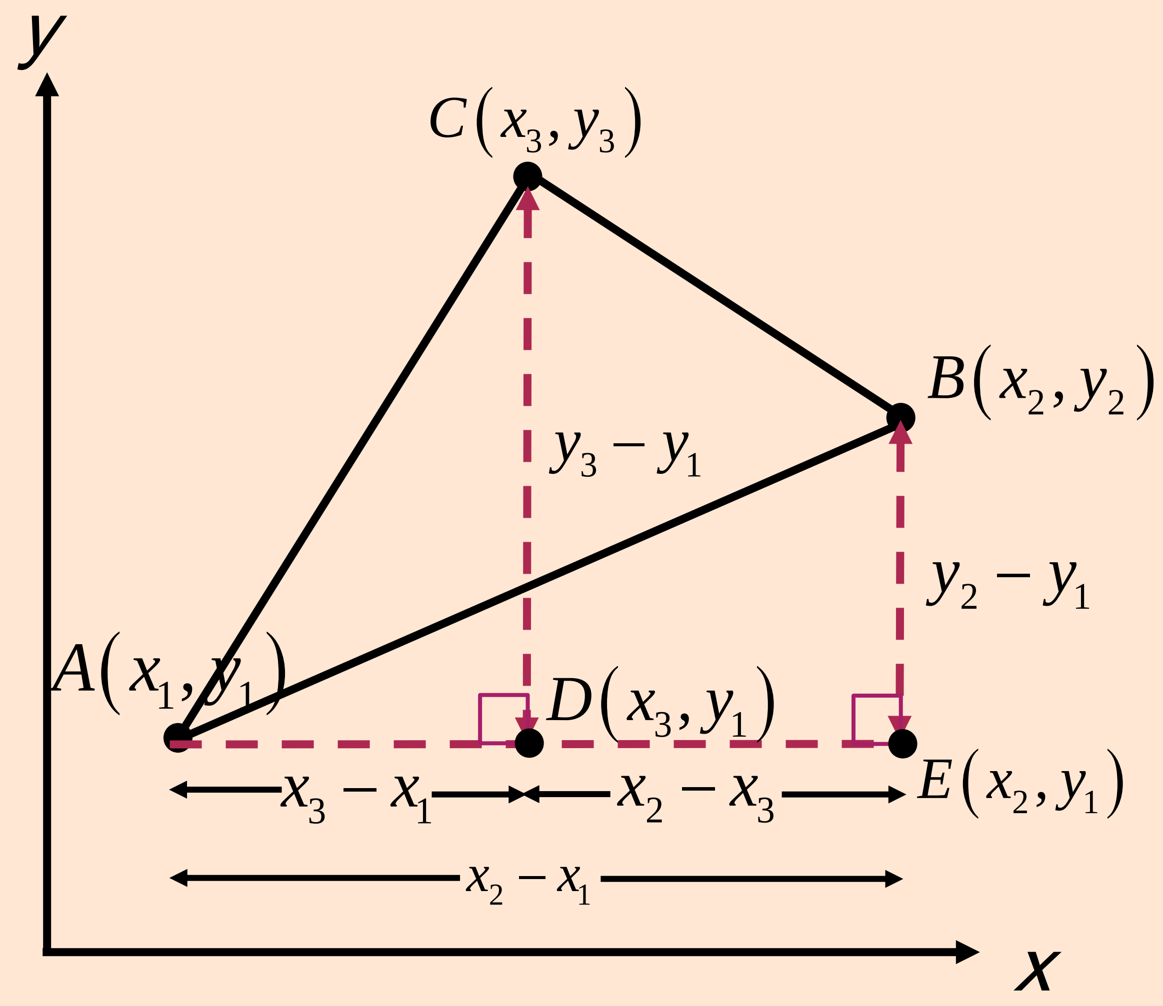
Shoelace algorithm

Area of ΔABC

$$= \frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix}\right|$$

$$= \frac{1}{2}\left|x_1y_2 + x_2y_3 + x_3y_1 - x_2y_1 - x_3y_2 - x_1y_3\right|$$

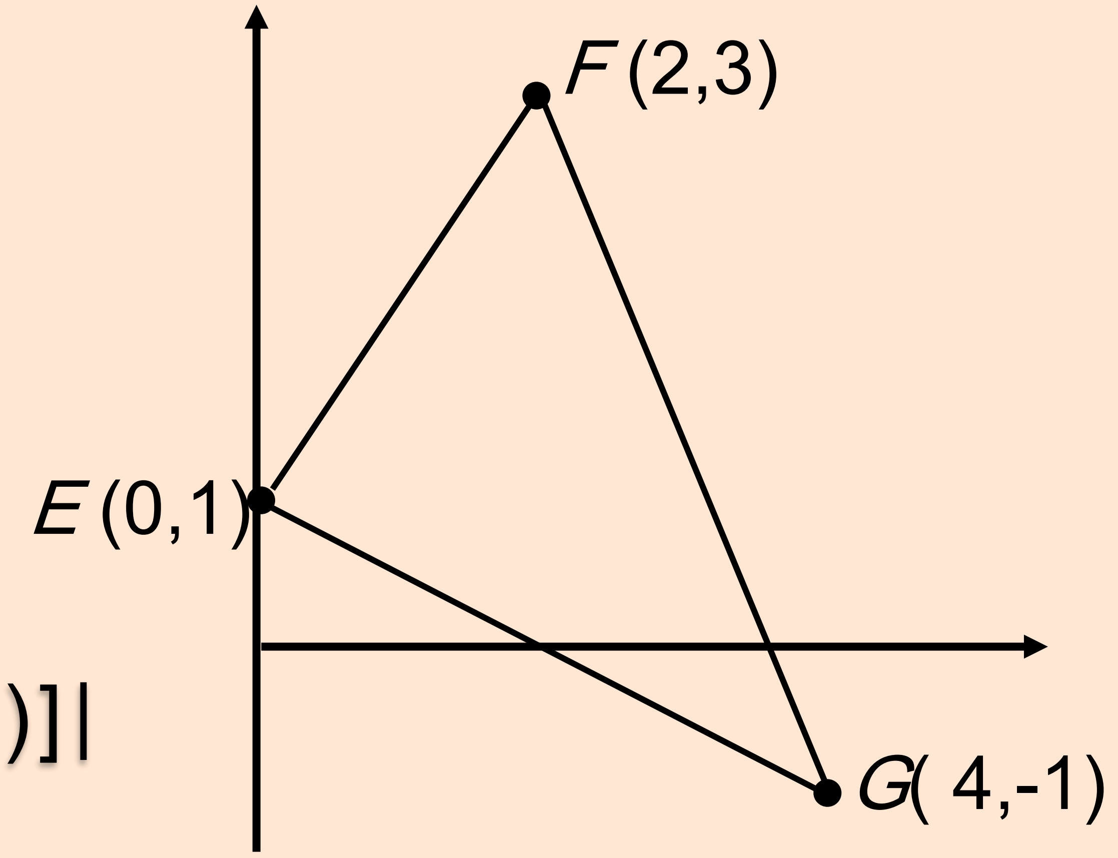
$$= \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right|$$



7.3.2 Determine the area of triangles by using the formula.

Example 13

Find the area of each of the following triangles with the given vertices.
E (0, 1), F (2, 3), G (4, –1)



Solution : Anti clockwise direction

Area of triangle EFG

$$= \frac{1}{2}\left|\begin{matrix} 4 & 2 & 0 & 4 \\ -1 & 3 & 1 & -1 \end{matrix}\right|$$

$$= \frac{1}{2}\,|[(4)(3) + (2)(1) + (0)(-1)] - [(2)(-1) + (0)(3) + (4)(1)]|$$

$$= \frac{1}{2}\,|(14) - (2)|$$

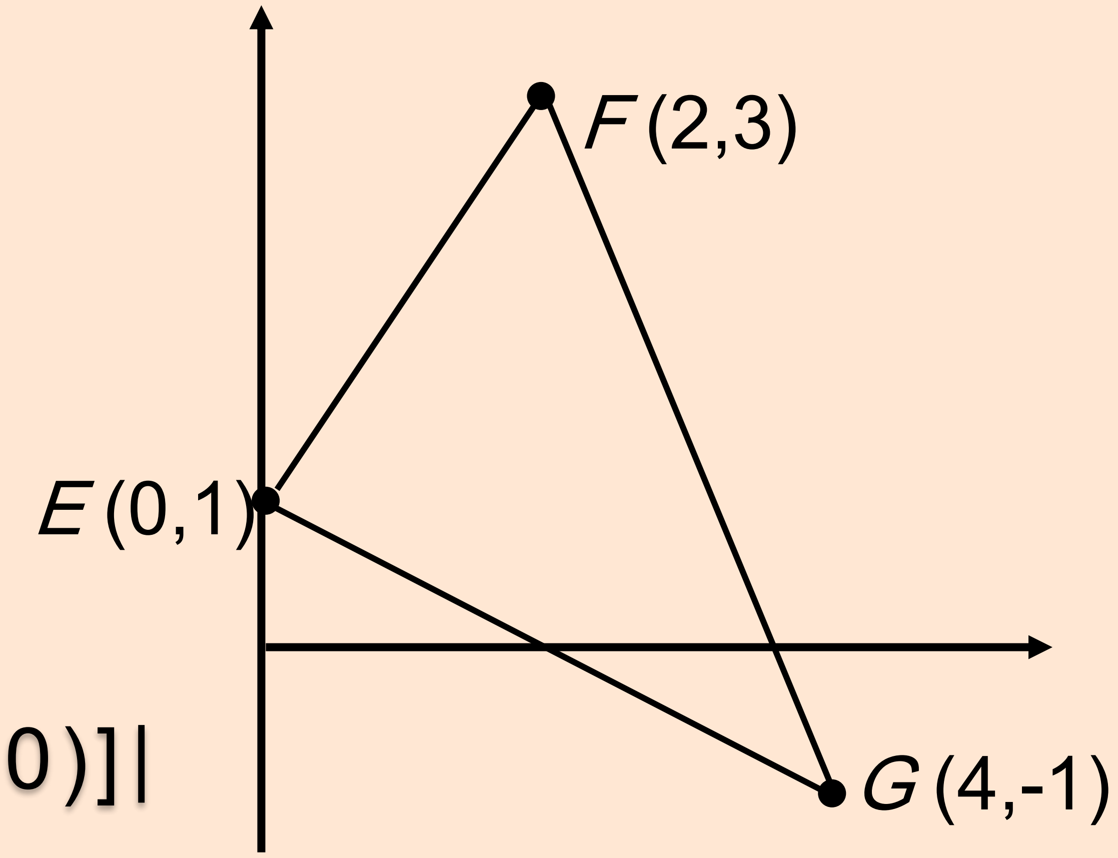
$$= \frac{1}{2}\,|12|$$

$= 6$ units^2

Example 13

Find the area of each of the following triangles with the given vertices.
$E\,(0, 1)$, $F\,(2, 3)$, $G\,(4, -1)$

Solution : Clockwise direction



Area of triangle EFG

$$= \frac{1}{2}\begin{vmatrix} 0 & 2 & 4 & 0 \\ 1 & 3 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\,|[(0)(3) + (2)(-1) + (4)(1)] - [(1)(2) + (3)(4) + (-1)(0)]|$$

$$= \frac{1}{2}\,|(2) - (14)|$$

$$= \frac{1}{2}\,|(-12)|$$

$= 6$ units^2

Example 13

Find the area of each of the following triangles with the given vertices.
E (0, 1), *F* (2, 3), *G* (4, –1)

Solution : Use Box Method

Step 1 : Find area rectangle

$4 \times 4 = 16$

Step 2 : Find area I, II and III

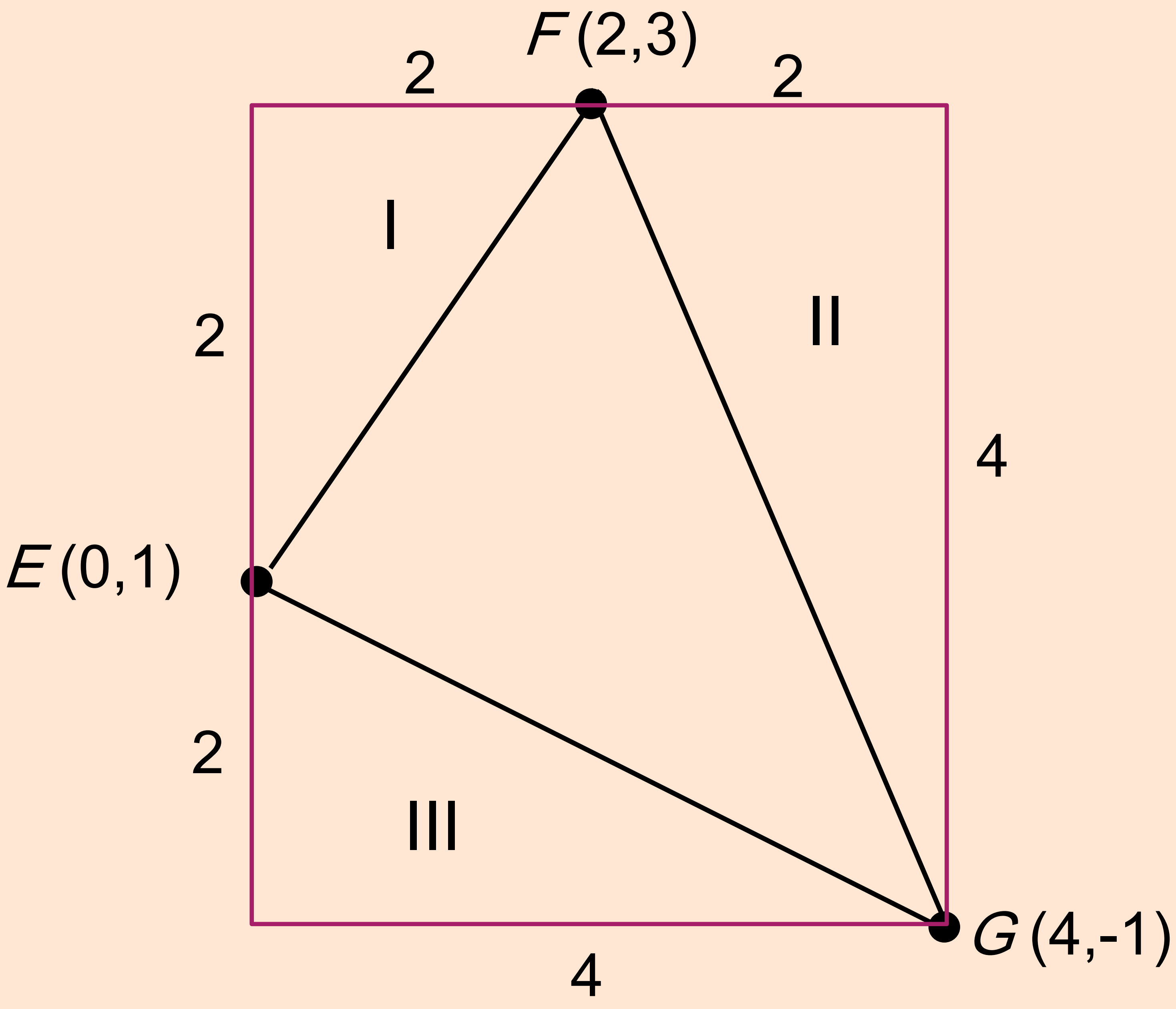
Area I $= \frac{1}{2}(2)(2) = 2$

Area II $= \frac{1}{2}(2)(4) = 4$

Area III $= \frac{1}{2}(2)(4) = 4$

Step 3 : Find area EFG = 16 – 2 – 4 – 4

= 6 units^2



Area of polygons

7.3.2 Determine the area of quadrilaterals by using the formula.

Area quadrilateral $ABCD$

$$= \frac{1}{2}\left|\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix}\right|$$

$$= \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\right|$$

Example 14

Given that A (–1,4), B (5,3), C (6,–1), D (0,3) are the vertices of quadrilateral $ABCD$. Find the area of quadrilateral $ABCD$.

Solution:-

Area of quadrilateral $ABCD$

$$= \frac{1}{2}\left|\begin{matrix} 0 & 6 & 5 & -1 & 0 \\ 3 & -1 & 3 & 4 & 3 \end{matrix}\right|$$

$$= \frac{1}{2}\left|[(0)(-1) + (6)(3) + (5)(4) + (-1)(3)] - [(6)(3) + (5)(-1) + (-1)(3) + (0)(4)]\right|$$

$$= \frac{1}{2}\left|[0 + 18 + 20 + (-3)] - [18 + (-5) + (-3) + 0]\right|$$

$$= \frac{1}{2}\left|35 - 10\right|$$

$$= \frac{1}{2}\left|(25)\right|$$

$= 12.5$ units^2

Area of polygons

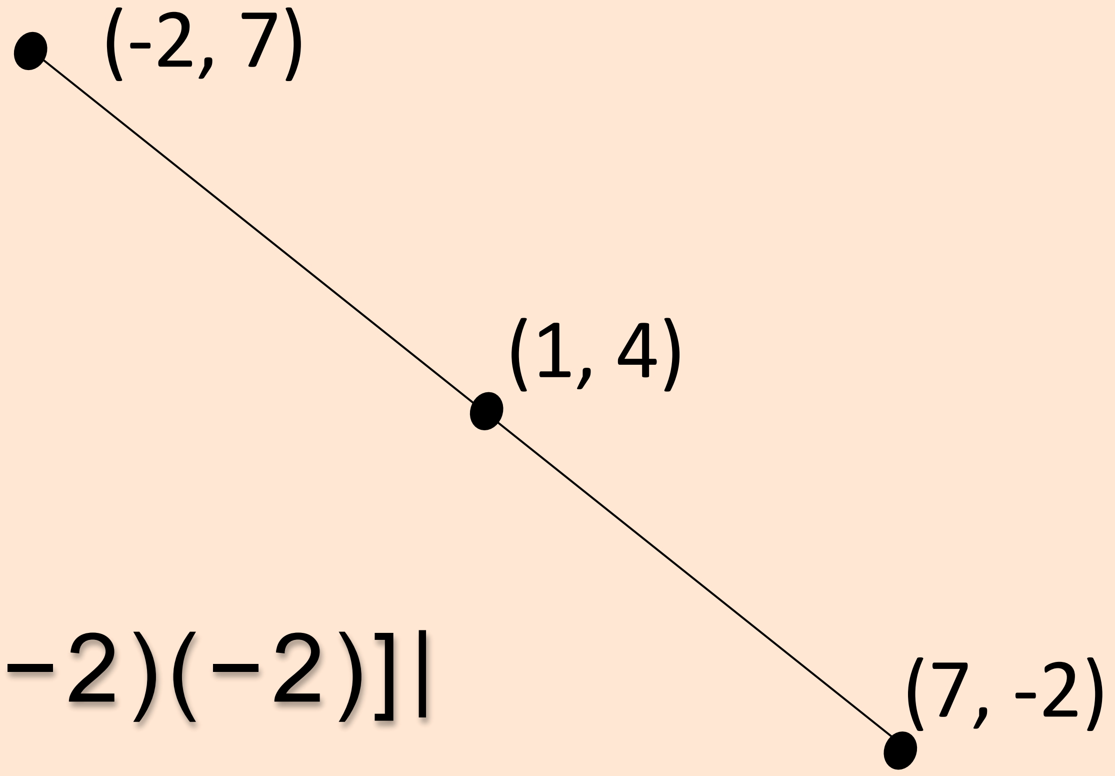
7.3.4 Make generalisation about the formula of area of polygons when the coordinates of each vertex are known, and hence use the formula to determine the area of polygons.

Area of polygon

$$= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & \dots & x_n & x_1 \\ y_1 & y_2 & \dots & y_n & y_1 \end{vmatrix}$$

Example 15

Determine whether the points (–2, 7), (1, 4) and (7, –2) are collinear or not.



Solution:-

Area

$$= \frac{1}{2}\begin{vmatrix} -2 & 1 & 7 & -2 \\ 7 & 4 & -2 & 7 \end{vmatrix}$$

$$= \frac{1}{2}\left|[(-2)(4) + (1)(-2) + (7)(7)] - [(7)(1) + (4)(7) + (-2)(-2)]\right|$$

$$= \frac{1}{2}\left|[-8 + (-2) + 49] - [7 + 28 + 4]\right|$$

$$= \frac{1}{2}\left|(39) - (39)\right|$$

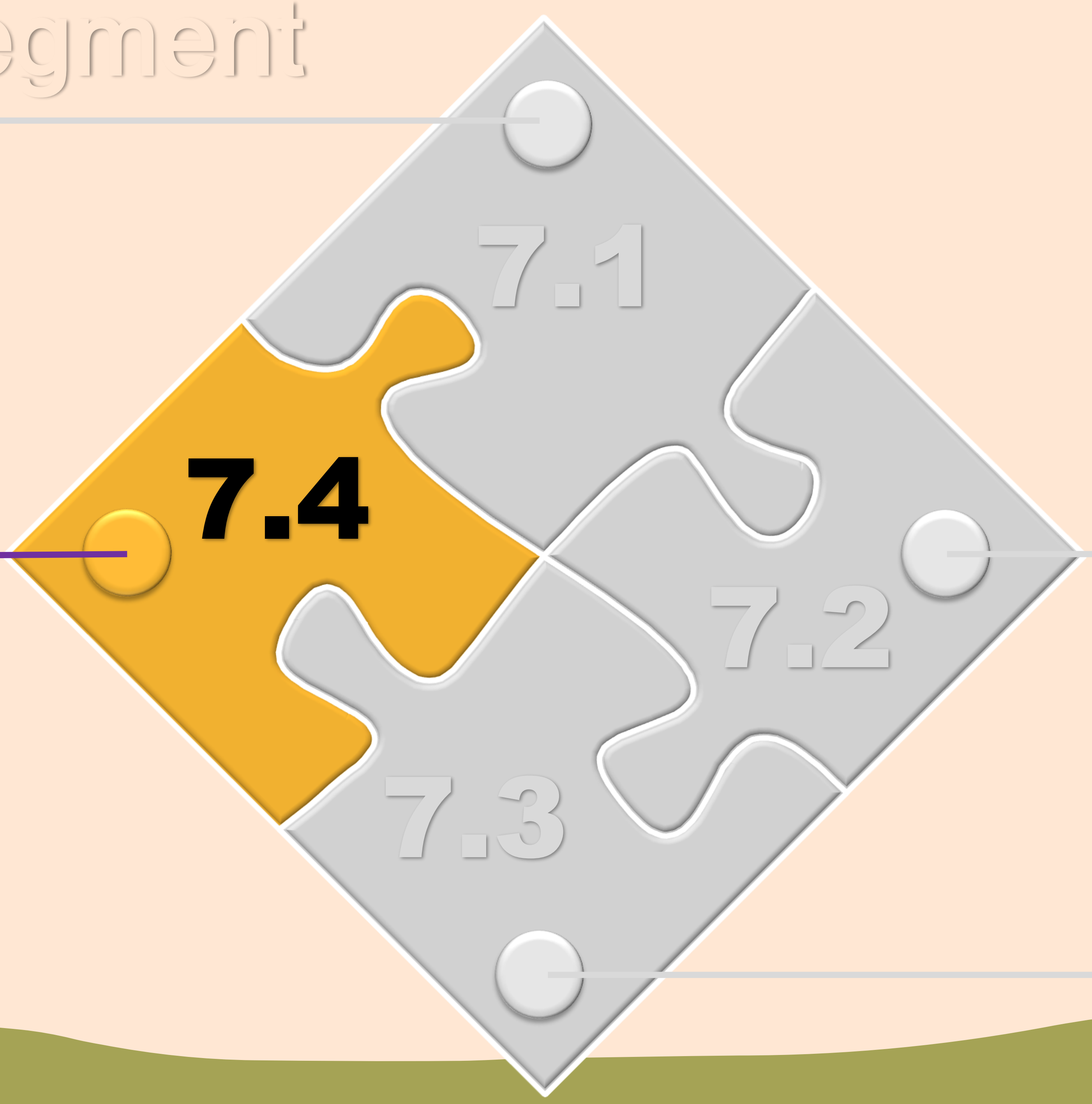
$$= 0$$

The area = 0, points (–2,7), (1,4), (7,-2) are collinear.

7. Coordinates Geometry

Divisor of a line segment

Equations of loci



Parallel and perpendicular lines

Area of polygons

Equations of Loci

PRIOR KNOWLEDGE : FORM 3
LOCI
8.2 Loci in Two Dimensions

7.4.1: Represent graphically, the locus that satisfies these conditions:-

(i) the distance of a moving point from a fixed point is constant,

(ii) the ratio of a moving point from two fixed points is constant,

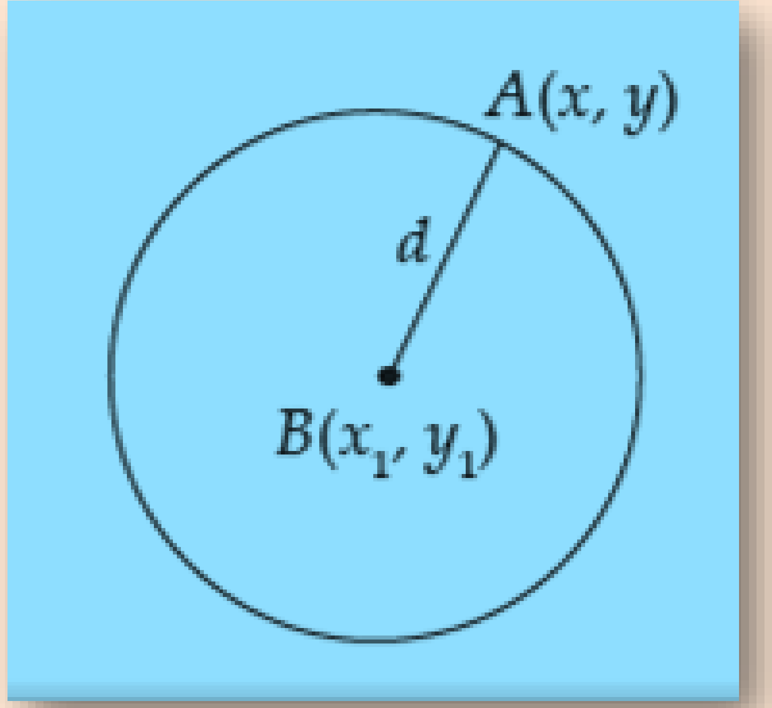
and hence determine the equation of the locus.

Equations of Loci

7.4.1 Determine the equation of the locus.

(i) Locus of a moving point from a fixed point is constant.

The equation of the locus of point $A\,(x, y)$ which moves such that its distance is d units from a fixed point $B\,(x_1, y_1)$:



$$(x - x_1)^2 + (y - y_1)^2 = d^2$$

Equations of Loci

Example 16

Find the equation of the locus of point $L(x, y)$ which moves such that its distance is 2 units from a fixed point $P(-5,0)$.

Solution:-

$$L(x, y)\ ,\ P(-5,0)$$

$$LP = 2$$

$$[x-(-5)]^2 + (y-0)^2 = 2^2$$

$$(x+5)^2 + (y)^2 = 2^2$$

$$x^2 + 10x + 25 + y^2 - 4 = 0$$

$$x^2 + y^2 + 10x + 21 = 0$$

Equations of Loci

7.4.1 Determine the equation of the locus.

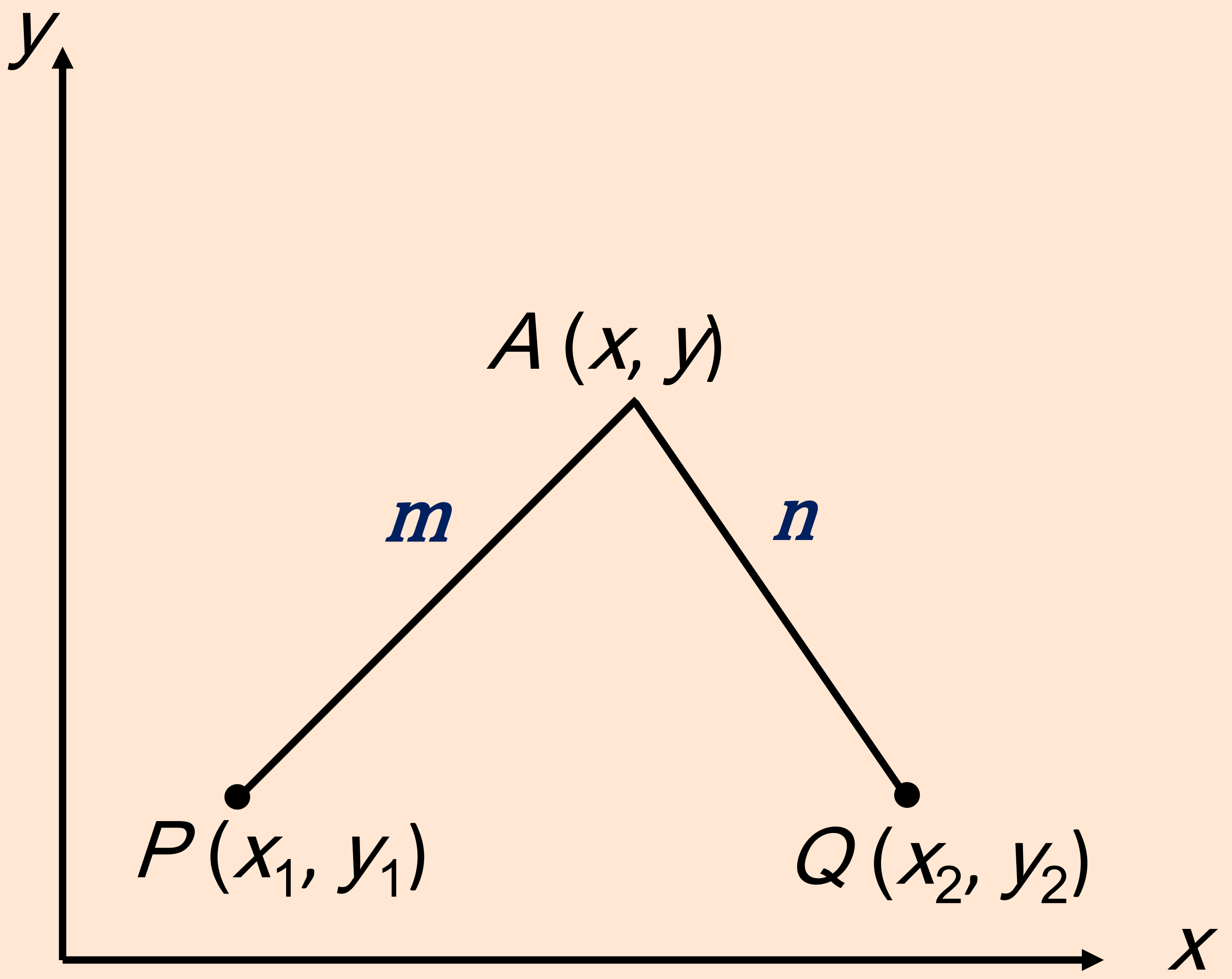
(ii) Locus of a moving point from two fixed point is constant.

The equation of the locus of point $A\ (x,\ y)$ which moves such that its distance from point $P\ (x_1, y_1)$ and point $Q\ (x_2, y_2)$ are in the ratio of $m : n$

$$\frac{AP}{AQ} = \frac{m}{n}$$

$$\frac{\sqrt{(x - x_1)^2 + (y - y_1)^2}}{\sqrt{(x - x_2)^2 + (y - y_2)^2}} = \frac{m}{n} \quad or$$

$$n\sqrt{(x - x_1)^2 + (y - y_1)^2} = m\sqrt{(x - x_2)^2 + (y - y_2)^2}$$



Example 17

Given B (3,2) A (–2,–1) and P (x,y) is a moving point such that $AP : PB = 2 : 3$. Find the equation of locus P.

Solution:-

$$\frac{AP}{PB} = \frac{2}{3} \qquad 3AP = 2PB$$

$$3\sqrt{[x-(-2)]^2 + [y-(-1)]^2} = 2\sqrt{(x-3)^2 + (y-2)^2}$$

$$9[(x+2)^2 + (y+1)^2] = 4[(x-3)^2 + (y-2)^2]$$

$$9(x^2 + 4x + 4 + y^2 + 2y + 1) = 4(x^2 - 6x + 9 + y^2 - 4y + 4)$$

$$9(x^2 + 4x + y^2 + 2y + 5) = 4(x^2 - 6x + y^2 - 4y + 13)$$

$$9x^2 + 36x + 9y^2 + 18y + 45 = 4x^2 - 24x + 4y^2 - 16y + 52$$

$$5x^2 + 5y^2 + 60x + 34y - 7 = 0$$

Example 18

Determine whether the locus $x^2 + y^2 + 4x - 7y + 12 = 0$ intersects with the straight line $y = -x + 1$ or not.

Solution:-

On the line $y = -x + 1$,

$$x^2 + (-x + 1)^2 + 4x - 7(-x + 1) + 12 = 0$$

$$x^2 + x^2 - 2x + 1 + 4x + 7x - 7 + 12 = 0$$

$$2x^2 + 9x + 6 = 0$$

$$b^2 - 4ac = (9)^2 - 4(2)(6)$$

$$= 33$$

$b^2 - 4ac > 0$ therefore the locus intersects with the straight line $y = -x + 1$ at two distinct points.

Kertas 1 SPMRSM 2017

Point *P* moves such that its distance from point *A*(5,0) is always 3 units.

(a) Find the equation of locus *P*.

(b) Hence , find the coordinates of the points where the locus of *P* intercept the *x*-axis.

[4 marks]

Solution :-

(a)

$$PA = 3$$

$$\sqrt{(x-5)^2 + y^2} = 3$$

$$x^2 - 10x + 25 + y^2 = 3^2$$

$$x^2 + y^2 - 10x + 16 = 0$$

(b) Intercept *x*-axis, *y* = 0

$$x^2 + (0) - 10x + 16 = 0$$

$$x^2 - 10x + 16 = 0$$

$$(x-2)(x-8) = 0$$

$$x = 2 \quad , \quad x = 8$$

$$(2\,,0)\ ,\ (8\,,0)$$

Kertas 2 Sec A SPMRSM 2017

Solution by scale drawing is not accepted.

Diagram 5 represents a science garden at MRSM Bestari. The garden is in quadrilateral shape and *BD* is a straight line.

The Science Club members built a reflexology path, *BD* and they want to build another straight reflexology path to join point *A* to path *BD* at point *M* (h, $2h$). The length of *AM is* $\sqrt{32}$ m.

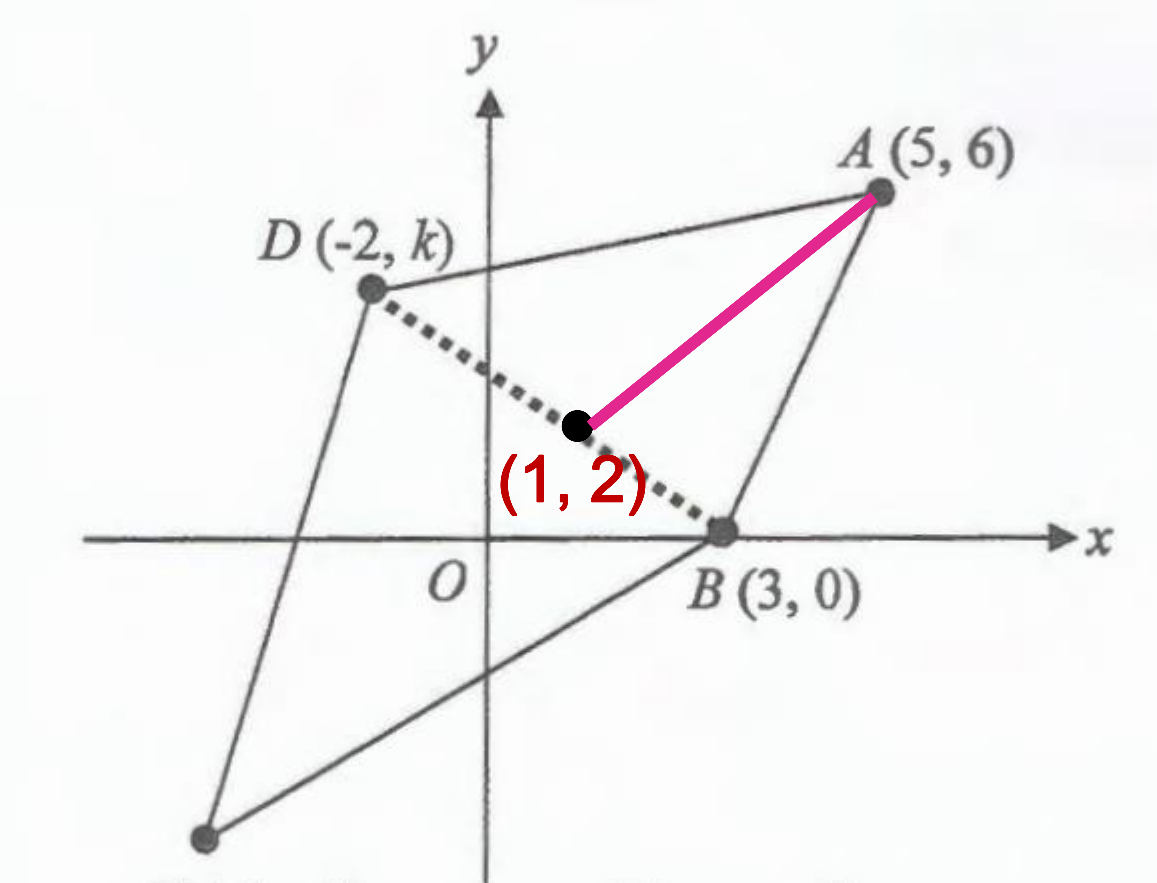


Diagram 5
Rajah 5

(a) Find the value of *h* and of *k*. [4 marks]

Solution:-

$$\sqrt{(h-5)^2+(2h-6)^2}=\sqrt{32}$$

$$5h^2-34h+29=0$$

$$(5h-29)(h-1)=0$$

$$h=\frac{29}{5},\ h=1$$

$$\frac{k-2}{-2-1}=\frac{2-0}{1-3}$$

$$k-2=3$$

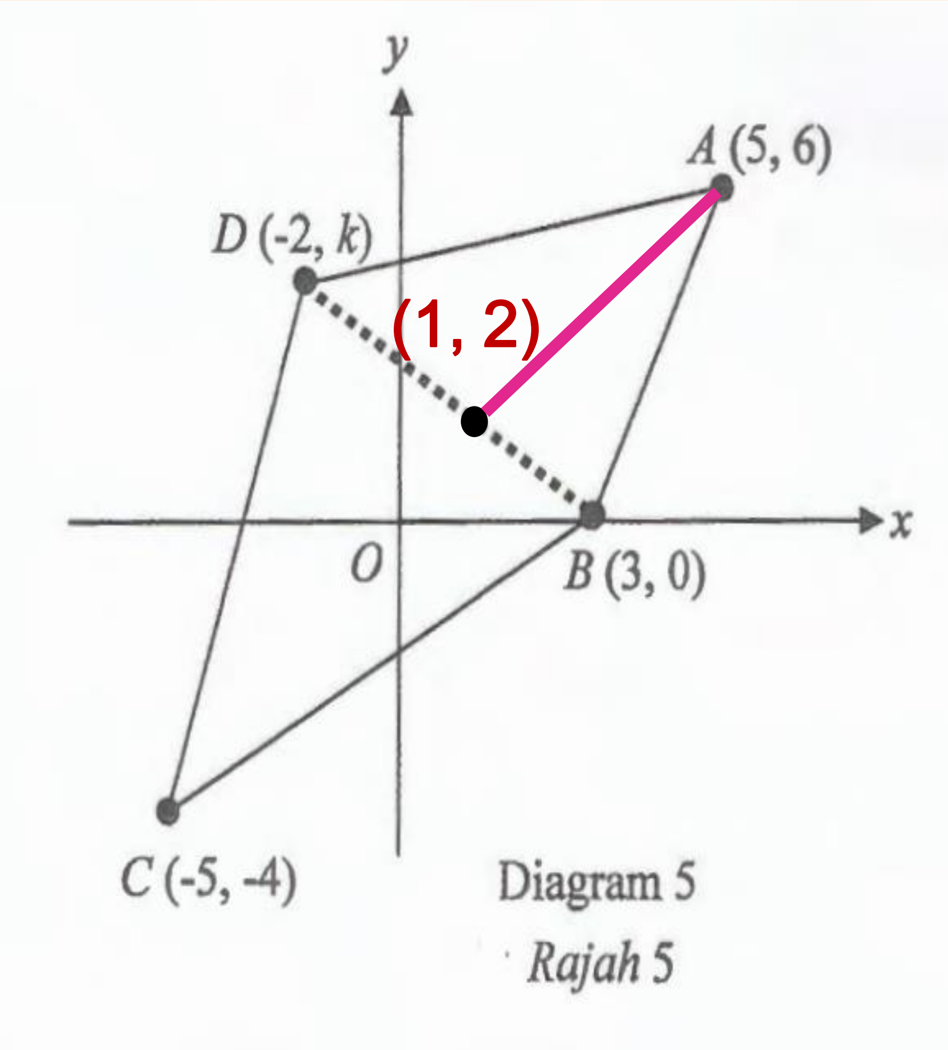
$$k=5$$

Kertas 2 Sec B SPMRSM 2017

(b) Hence,

(i) determine whether *M* is the shortest distance from *A* to *BD*,

(ii) find the area, in m^2 , of the science garden. [4 marks]



Solution:-

(i)

$$\left(\frac{6-2}{5-1}\right)\left(\frac{2-0}{1-3}\right)=\frac{4}{4}\left(\frac{2}{-2}\right)$$
$$=-1$$
$$m_{AM}.m_{DB}=-1$$

Therefore *M* is the shortest distance from *A* to *BD*

(ii)

Area

$$=\frac{1}{2}\begin{vmatrix}3 & 5 & -2 & -5 & 3\\ 0 & 6 & 5 & -4 & 0\end{vmatrix}$$
$$=\frac{1}{2}\left|\left[3(6)+5(5)+(-2)(-4)+(-5)(0)\right]-\left[0(5)+6(-2)+5(-5)+(-4)(3)\right]\right|$$
$$=\frac{1}{2}\left|51-(-49)\right|$$
$$=50$$

Kertas 2 Sec A SPMRSM 2020

Solution by scale drawing is not accepted.

Diagram 4 shows coordinates of point *A*, *B* and *C* on a Cartesian plane.

(a) Find the equation of a straight line which passes through point *C* and perpendicular to the straight line *AB*. [3 *marks*]

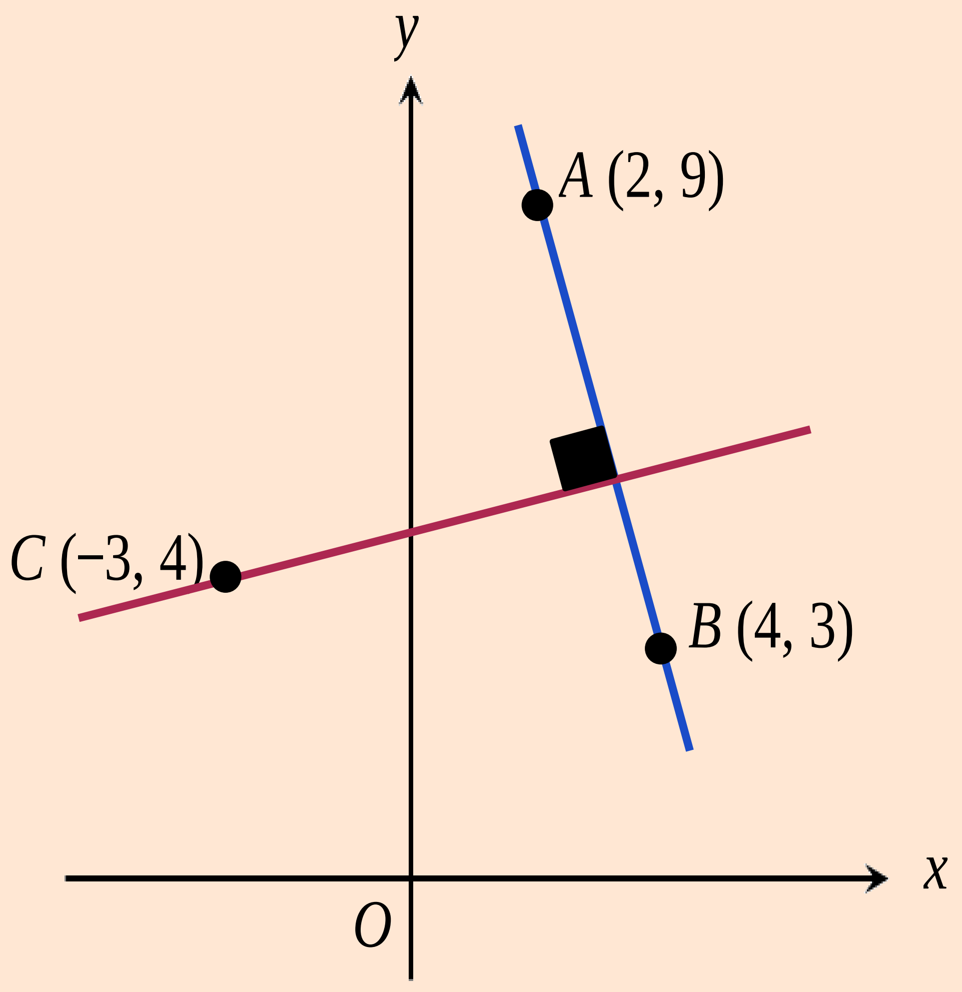


Diagram 4

Solution:-

(a) $m_{AB} = \frac{9-3}{2-4} = -3$

$-3m_2 = -1$

$m_2 = \frac{1}{3}$

$y - 4 = \frac{1}{3}(x + 3)$

$y = \frac{1}{3}x + 5$

Kertas 2 Sec A SPMRSM 2020

(b) (i) Point *P* moves such that its distance is always 5 units from the midpoint *AB*.

Find the equation of the locus *P*.

(ii) Hence, show that locus *P* does not intersects the *x*-axis. [5 *marks*]

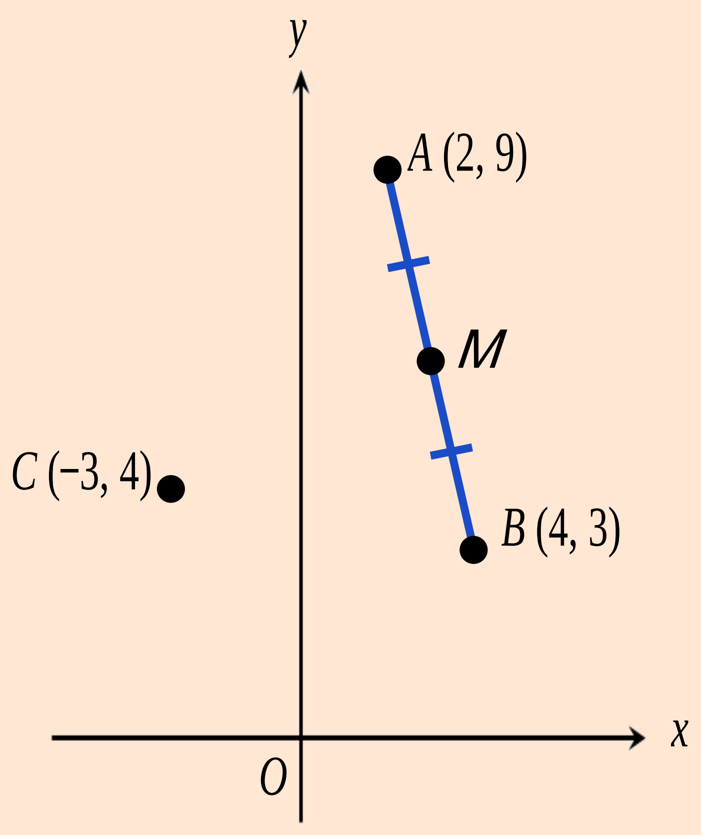


Diagram 4

Solution:-

(b) (i) $P(x, y)$

$$\text{Midpoint } AB,\ M = \left(\frac{2+4}{2}, \frac{9+3}{2}\right)$$

$$= (3,6)$$

$$PM = 5$$

$$\sqrt{(x-3)^2 + (y-6)^2} = 5$$

$$x^2 + y^2 - 6x - 12y + 20 = 0$$

(ii)

$$x^2 + (0)^2 - 6x - 12(0) + 20 = 0$$

$$x^2 - 6x + 20 = 0$$

$$b^2 - 4ac = (-6)2 - 4(1)(20)$$

$$= -44$$

$b^2 - 4ac < 0$ therefore the locus *P* does not intersect the *x*-axis

Kertas 2 Sec B SPMRSM 2018

Solution by scale drawing is not accepted.
Diagram 8 shows the straight line *RQ* which is perpendicular to the straight line *KL* at point *Q*.

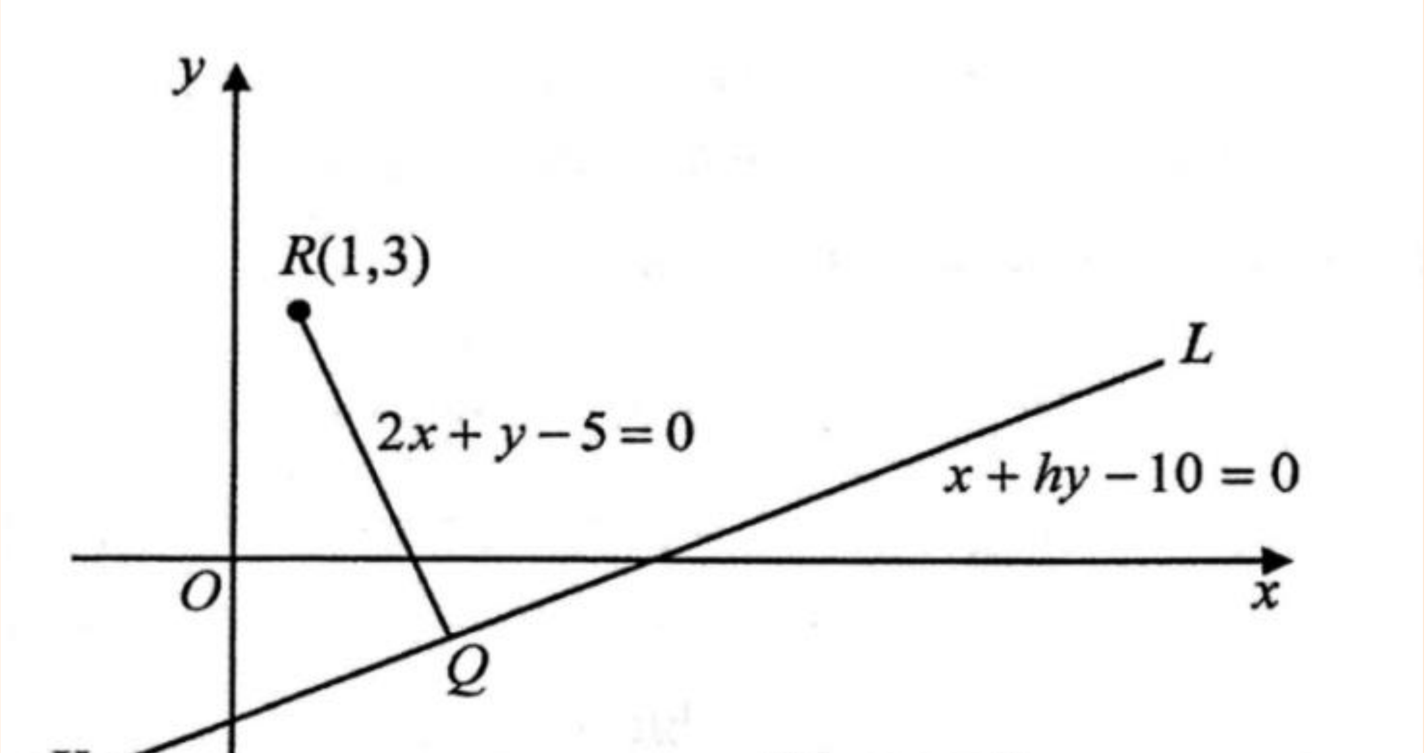


Diagram 8

(a) Find
(i) the value of *h*,
(ii) the coordinates of *Q*. [4 marks]

Solution:-

(a)(i) $m_{RQ}.m_{KL} = -1$

$y = -2x + 5$

$m_{RQ} = -2$

$y = -\frac{1}{h}x + \frac{10}{h}$

$-2\left(-\frac{1}{h}\right) = -1$

$\text{h} = -2$

(ii) $-2x + 5 = -\frac{1}{(-2)}x + \frac{10}{(-2)}$

$4x - 10 = -x + 10$

$5x = 20$

$x = 4$

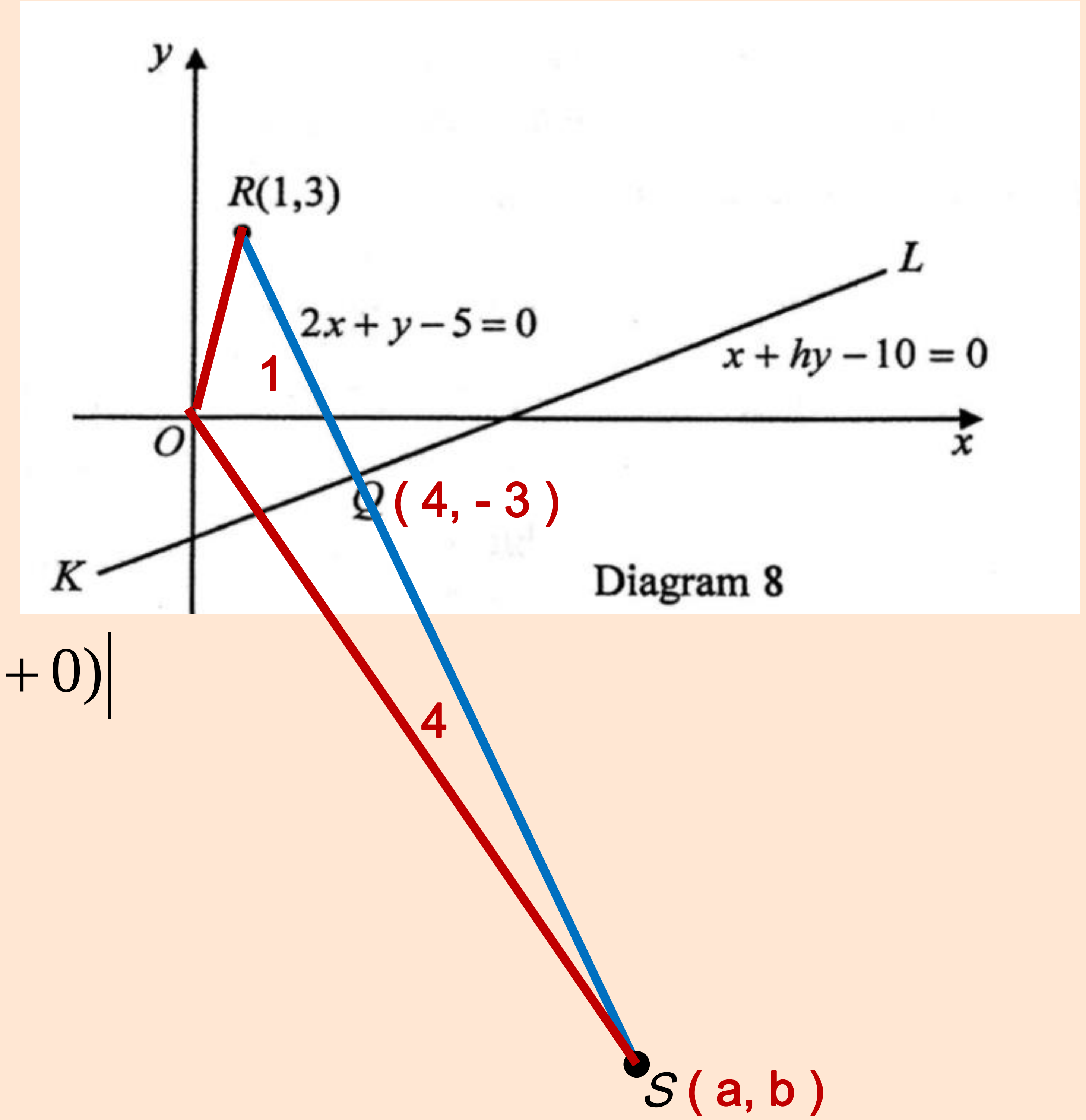
$y = -2(4) + 5$

$= -3$

$\therefore Q\ (4, -3)$

Kertas 2 Sec B SPMRSM 2018

(b) The straight line *RQ* is extended to *S* such that $RQ : RS = 1 : 5$.
Find the area, in $unit^2$, of triangle *ROS*. [4 marks]



Solution:-

(b) $\dfrac{4(1)+1(a)}{1+4} = 4$

$a = 16$

$\dfrac{4(3)+1(b)}{1+4} = -3$

$b = -27$

$\therefore S(16, -27)$

Area *ROS*

$$= \frac{1}{2}\begin{vmatrix} 0 & 16 & 1 & 0 \\ 0 & -27 & 3 & 0 \end{vmatrix}$$

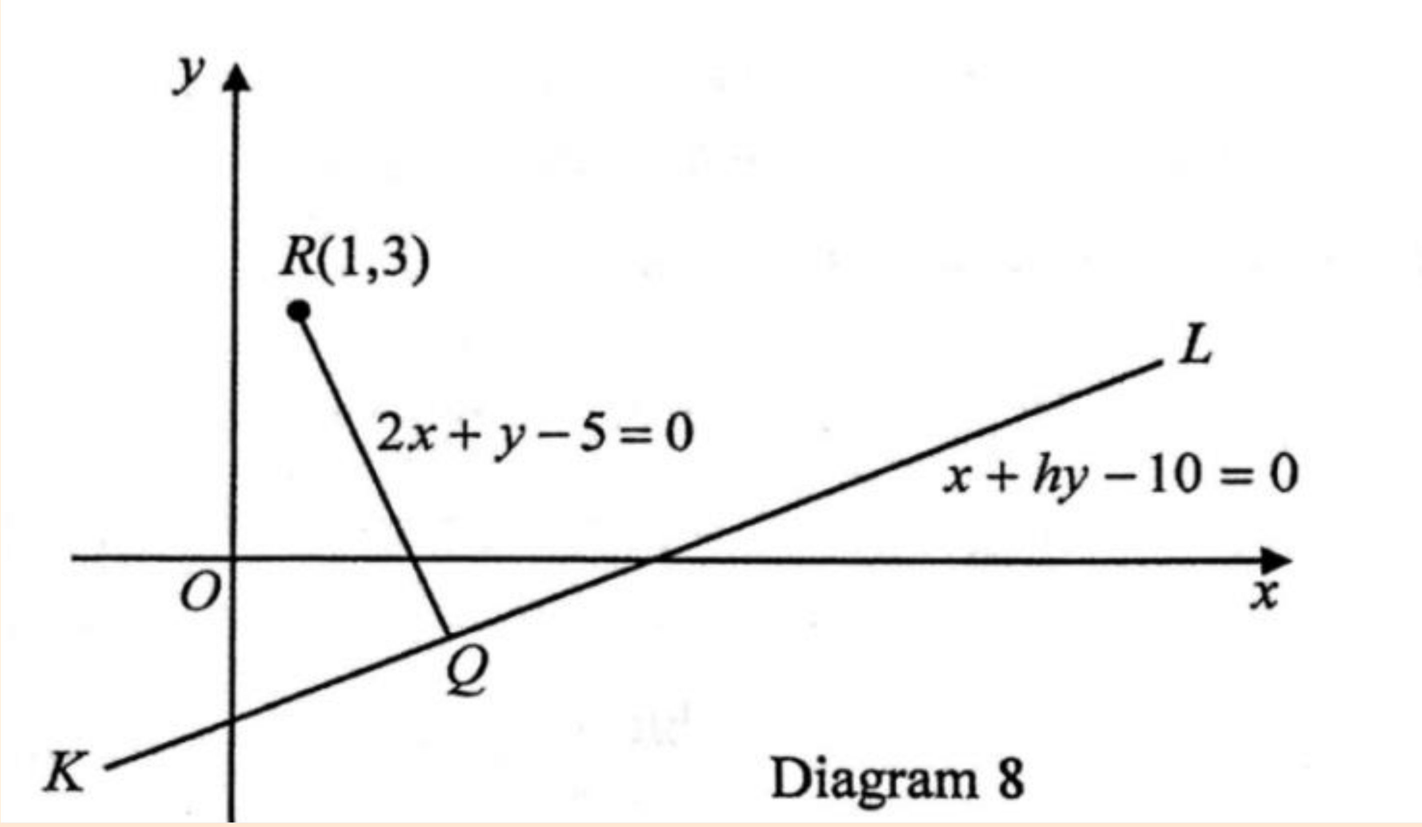
$$= \frac{1}{2}\left|(0+16(3)+0)-(0+(-27)(1)+0)\right|$$

$$= \frac{1}{2}\left|48+27\right|$$

$$= \frac{75}{2} \ @\ 37.5$$

Kertas 2 Sec B SPMRSM 2018

(c) A point T moves such that its distance from point R is always 5 units.
Find the equation of the locus of T. [2 marks]



Solution:-

(c) $TR = 5$

$$\sqrt{(x-1)^2+(y-3)^2}=5$$

$$x^2-2x+1+y^2-6y+9=5^2$$

$$x^2+y^2-2x-6y-15=0$$

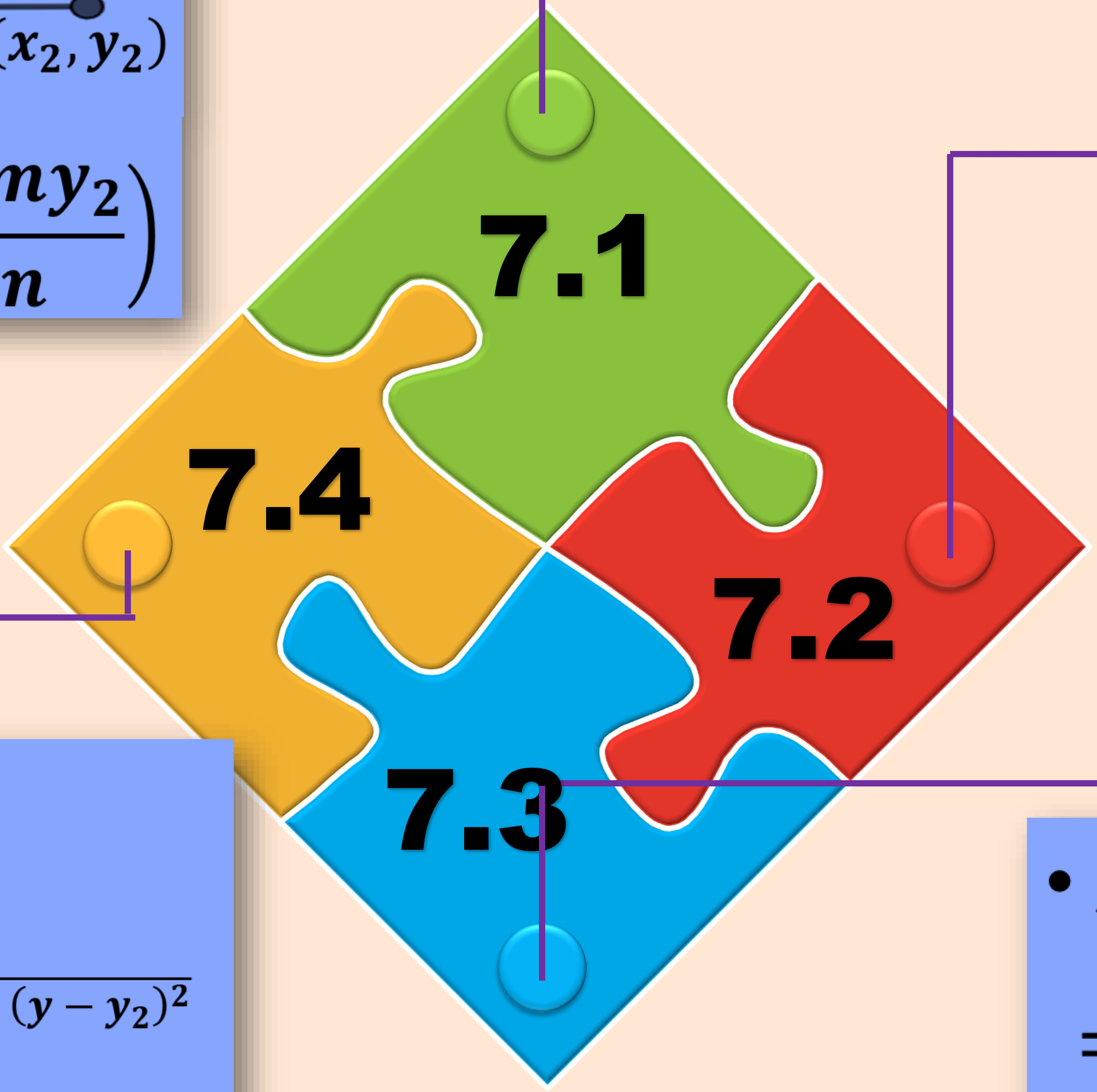
Summary of Coordinates Geometry

Divisor of a line segment

m n

(x_1, y_1) P (x_2, y_2)

$$P = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}\right)$$



Parallel and perpendicular lines

- **Parallel lines :**
  $m_1 = m_2$
- **Perpendicular lines:**
  $m_1 \times m_2 = -1$

Equations of loci

- **Constant distance from a fixed point:**
  $\sqrt{(x - x_1)^2 + (y - y_1)^2} = d$
- **Distance from two points in the ration m : n:**
  $n\sqrt{(x - x_1)^2 + (y - y_1)^2} = m\sqrt{(x - x_2)^2 + (y - y_2)^2}$
- **Equidistant from two points:**
  $\sqrt{(x - x_1)^2 + (y - y_1)^2} = \sqrt{(x - x_2)^2 + (y - y_2)^2}$

Area of polygons

- **Area**

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & \dots & x_n & x_1 \\ y_1 & y_2 & y_3 & \dots & y_n & y_1 \end{vmatrix}$$